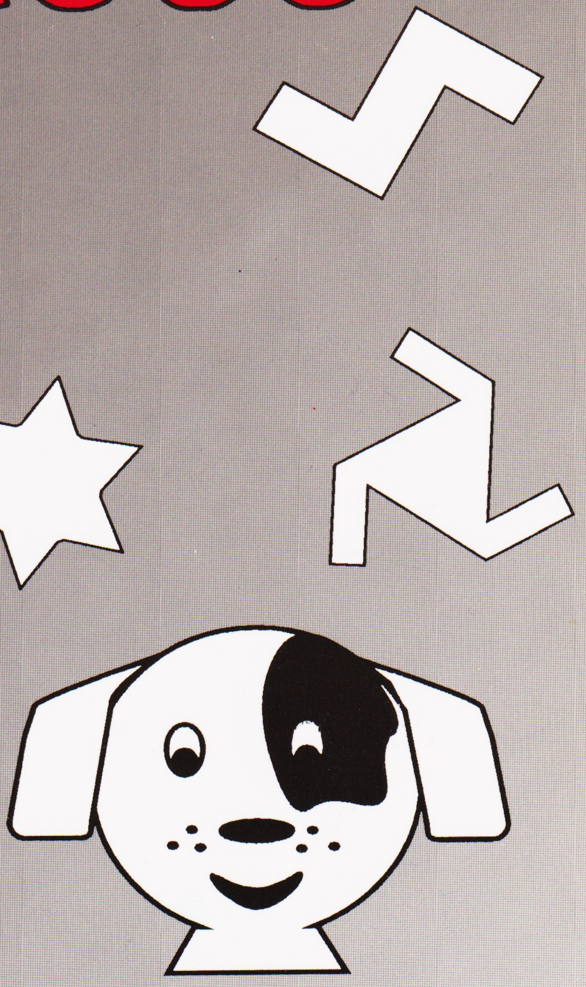
Maths through Databases



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Mathematics *through* Databases

By Roger Keeling
and Senga Whiteman

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Mathematics through Databases

Introduction

There is a variety of information retrieval programs available for use by primary and secondary school pupils and, because of their experiences with this software, most teachers are becoming more confident in their ability to manage an information handling activity, although providing the appropriate degree of challenge can be more difficult. The starting points for information handling activities can be many and varied. Young children can collect data about themselves and make comparisons between their shoe size and height or between their type of footwear and the season. Older children can collect similar data from a project on 'Ourselves' in order to look for relationships between certain measurements or to support, or more often disprove, a hypothesis about proportions in the human body (eg. Is arm length related to the size of hand span?). Children can collect data as the result of a survey, for example, to find the tastiest crisp, the best value washing powder or the most popular childrens' author. These activities are well documented in journals and books relating to the educational value of IT.

Although there are lots of areas to explore, teachers need to ensure that each time a child is involved in information handling the challenges increase and the associated skills are continually developed. This publication is designed to widen the horizons for information retrieval in both primary and lower secondary schools and to offer some ideas about how to increase the sophistication, and/or the complexity, of information handling activities. (Definitions: data is what is stored on the computer (words, numbers etc), while information handling is what the human brain does when it starts processing the data; interpreting, analysing etc.)

Information handling activities can focus on datafiles which have been prepared in advance, or they can include the collection of appropriate data by the children themselves. If the children are initiating an investigation from scratch they may have an opportunity to develop a wider range of skills, involving both the creation and interrogation of a database. On the other hand, it may be that exploring an existing file provides an opportunity to explore data in more depth and to refine their interrogation techniques and the formulation of hypotheses. Whatever the source, data can be examined in order to identify a specific example, to look for a pattern or to investigate a hypothesis. Children need an opportunity to participate in activities which involve exploring data they have collected as well as data from other sources.

Using this publication

There are four activities included in this book. The first two, 'Find the "real" Father Christmas' and 'Who stole the sausages?', have been devised to introduce children to interrogating imaginary, pre-prepared, databases. They are introductory activities and, thus, are suitable for children at the upper end of key stage 1 or the lower end of key stage 2 (depending upon previous information handling experiences). They can also be used for older children with special needs who enjoy the challenge of detective games. Each activity is based upon identifying a specific example from others in the set. The identification is based upon clues given in a series of scenarios. The illustrations are based upon the idea of an identikit. They are not meant to be realistic, but to show the presence, or absence, of specific features.

The 'Father Christmas' and 'Dogs' activities also lend themselves to work involving branching databases (eg. Sorting Game, Branch, Animal, Routes). A typical tree generated as a result of using Sorting Game is included at the end of the 'Dogs' section to give a feel for what such a tree could look like.

These first two activities are learning games. They are designed to supplement a range of explorations, starting from the concrete and progressing to the abstract. They have been included here because they provide a "fun" way to introduce information handling to both teachers and children. They provide an experience which becomes progressively more challenging. Teachers who would like to explore the possibilities of information handling can try these kinds of detective games with their children and use them as a spring-board to investigations devised by themselves. We acknowledge that these activities are based on fiction. There could be an element of "so-what" about discovering the solutions. However, in practice, we have discovered that they are motivating, unthreatening and provide a useful starting point for exploring one aspect of database use. The collection of genuine data can offer children challenging learning experiences - perhaps these games can provide a starting point for scenarios, created by children, based on people in their school, their favourite soft toy or characters from published fact or fiction.

The second pair of activities focus on mathematical investigations. 'Puzzling Shapes' is a key stage 2 activity while 'Number Investigations' is designed for children at the upper end of key stage 2 and at key stage 3. These activities use pre-prepared databases that enable children to focus on mathematical concepts and to develop their thinking in terms of both shape and number work. In particular they are designed to encourage children to collaborate and to discuss mathematical ideas. Developing an appropriate vocabulary, recognising the attributes of shapes, looking for patterns and formulating hypotheses are all key concepts in these two activities.

Each of the four activities is accompanied by the appropriate Grass datafile. A full printout of each database in included in the pack, with the exception of the 'Number' database. Hence if you wish to use a different 'host' database you will need to retype the data - it will probably take less than half an hour for each database. The 'Number' database is more substantial (900 records); if you wish to use this in a different format, please contact us and we may be able to transfer it for you.

The illustrations of the Father Christmases, dogs and shapes are all photocopiable. They have been reproduced as part of this book simply to ensure that the 'set' is kept together - if a workcard goes missing, it will spoil the whole activity. In the case of 'Puzzling Shapes' and 'Number Investigations' a selection of answers have been provided where appropriate.

The ideas in the book provide starting points. We know that teachers will generate further ideas once they have a feel for the potential work that can be achieved from each database. The work associated with these activities is closely mapped to the National Curriculum. In each case reference has been made to those Attainment Targets that the activities address.

Information Handling and the Maths National Curriculum

Attainment Target 1:

Using and applying mathematics requires children to talk about their work and ask questions. At level 2 children can describe their work and check results and ask and respond to questions along the lines of "What would happen if?". At level 3 children can explain their work and record their findings systematically; they can investigate and test predictions and check their results and consider whether or not they are sensible. Children at level 4 can record their findings and present them in oral, written or visual form; make generalisations or simple hypotheses. Children at level 5 can interpret mathematical information presented in oral, written or visual form.

Attainment Target 5:

Handling data requires children at level 2 to construct and interpret block graphs for discrete data; choose criteria for sorting and classifying objects and record results. Children at level 3 can extract specific pieces of information from tables and lists; access information in a simple database; construct and interpret barcharts and graphs. Children at level 4 can insert, interrogate and interpret data in a computer database; construct and interpret bar and line graphs and frequency diagrams. Children at level 5 can insert and interrogate data in a computer database; drawing conclusions; construct and interpret pie charts.

Impostors (Find the "real" Father Christmas)

There are 24 Father Christmases but only one fits the description of the "real" Father Christmas. Each Father Christmas has one record on the database. The field names, and the possible entries, are shown below.

Name:	
Moustache:	wide or narrow
Button(s):	1 or 2
Footwear:	boots or shoes
Pom-pom:	yes or no
Sack:	left or right (upon which shoulder does he carry the sack?)

When this activity was devised originally, there were fields relating to eye colour, boot colour, and hair colour. As it is produced here in black and white these attributes have been replaced by others which do not require colour. However, the pictures could be coloured in and the datafile extended accordingly.

The illustrations of the Father Christmases are on the following pages. There is also a set of work-cards designed to introduce this activity to the children. These become progressively more difficult and the final task focuses on the identity of the "real" Father Christmas.

Activity card 1 suggests some graphs to explore.

Activity cards 2 and 3 suggest simple searches which will familiarise the children with the data.

Activity card 4 describes the characteristics of the "real" Father Christmas.

Suggested extension activities:

1) Increasing the attributes of each Father Christmas, perhaps to include colour of eyes etc., or other features devised by the children.

2) Introducing some Mother Christmases and deciding their identifying features (a moustache, whether wide or narrow, might not be relevant). This could lead to the redefinition of certain existing field names.

3) Calculating the number of possible Father Christmases that there could be, given the number of variables each has. (There is not a full set here, some are missing!)

4) Creating a file on a hierarchical database (Sorting Game (BBC), Branch (BBC, Nimbus), Routes (Archimedes)). The children would need a set of the pictures, with each Father Christmas named. They could formulate some likely questions which would identify different Father Christmases, and then, starting with two illustrations, type in a question which would have a "Yes" answer for one Father Christmas and a "No" answer for the other. (For example, "Does he have a moustache which is wider than his face?") They could then add further Father Christmases and build up the file with different questions, e.g. "Has his jacket got two buttons?", or "Is he wearing big black boots?".

5) Creating other scenarios relating to the exploits of Father Christmas, with clues built in to aid identification.

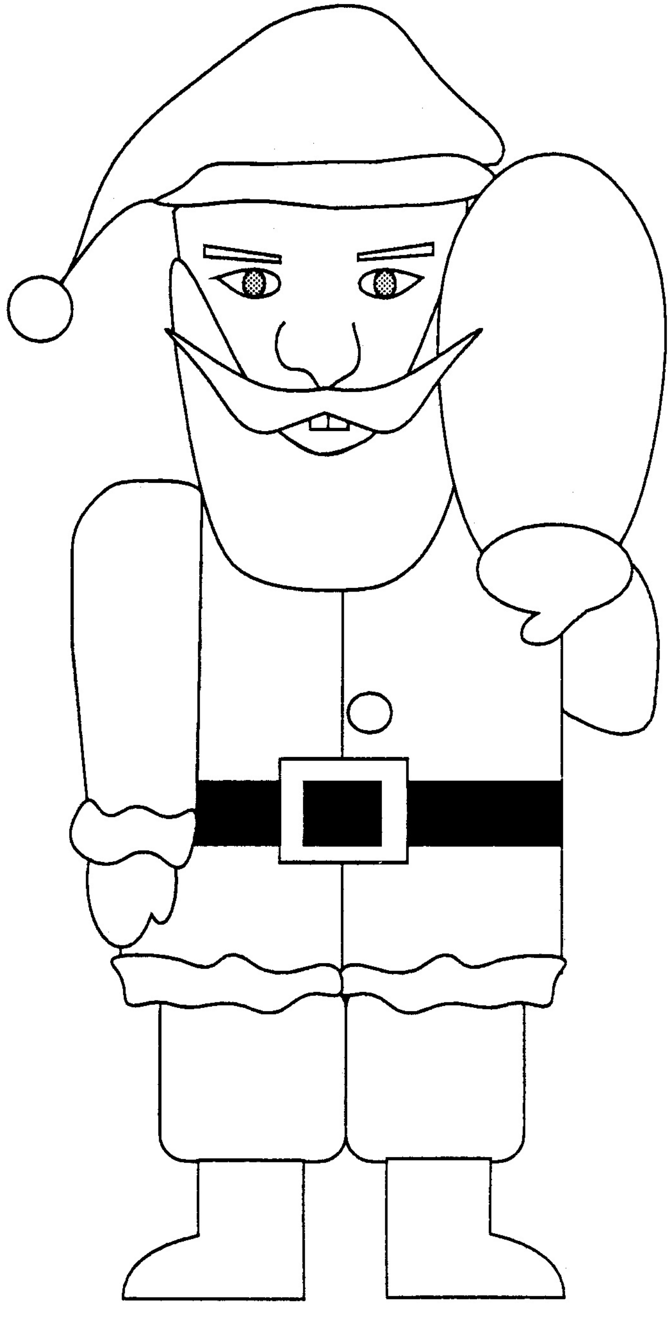
Here is a complete print out of the Father Christmas file:

Pictnum	Name	Moustache	Footwear	Pom-pom	Sack	Buttons
1	Waldemar	wide	boots	yes	left	1
2	Ulf	wide	boots	no	left	1
3	Gustav	narrow	boots	no	right	1
4	Caspar	narrow	boots	yes	left	2
5	Fredrik	narrow	shoes	no	left	2
6	Magnus	narrow	shoes	yes	left	1
7	Jorgen	wide	boots	yes	right	1
8	Edvard	wide	shoes	no	left	2
9	Amund	narrow	boots	yes	right	2
10	Sigurd	wide	shoes	yes	right	1
11	Olaf	wide	shoes	yes	left	1
12	Nils	wide	boots	yes	left	2
13	Axel	wide	shoes	yes	right	2
14	Halvard	wide	boots	no	right	1
15	Ransu	narrow	shoes	no	left	1
16	Detlof	wide	boots	no	left	2
17	Torolf	wide	boots	yes	right	2
18	Elof	wide	shoes	yes	left	2
19	Lennart	wide	shoes	no	left	1
20	Birger	wide	boots	no	right	2
21	Kettil	wide	shoes	no	right	2
22	Gjord	narrow	boots	no	right	2
23	Algot	narrow	boots	no	left	1
24	Pietari	narrow	shoes	yes	left	2

An example record from the Father Christmas file.

Pictnum	3
Name	Gustav
Moustache	narrow
Footwear	boots
Pom-pom	no
Sack	right
Button(s)	1

1



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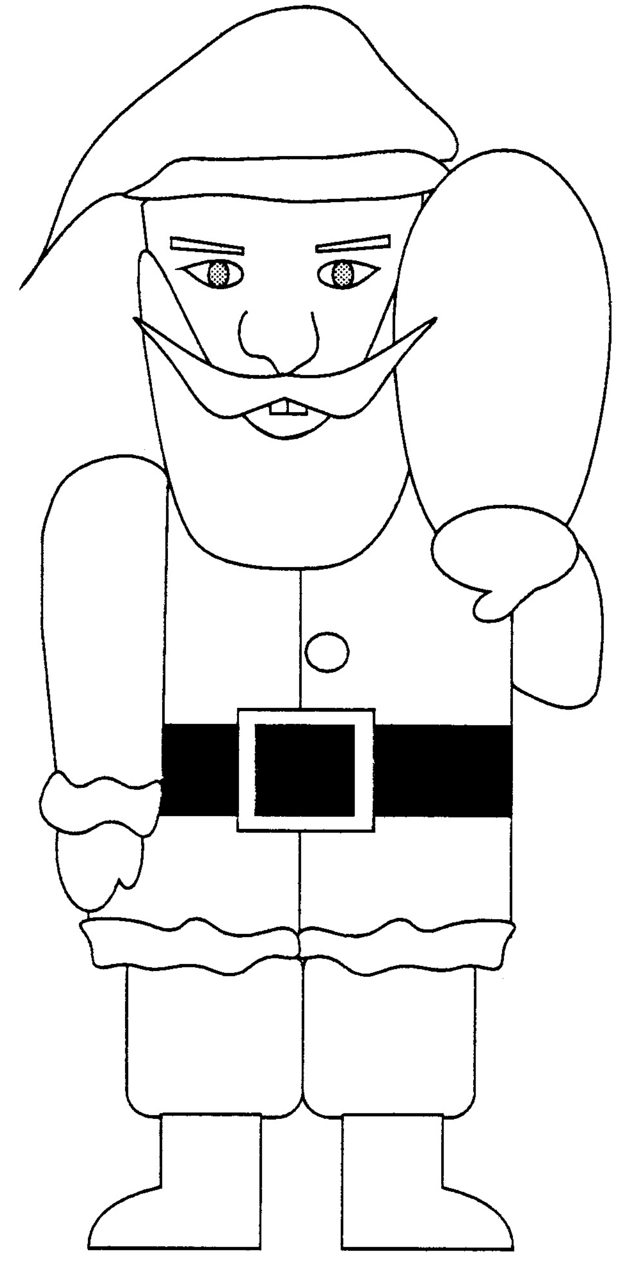

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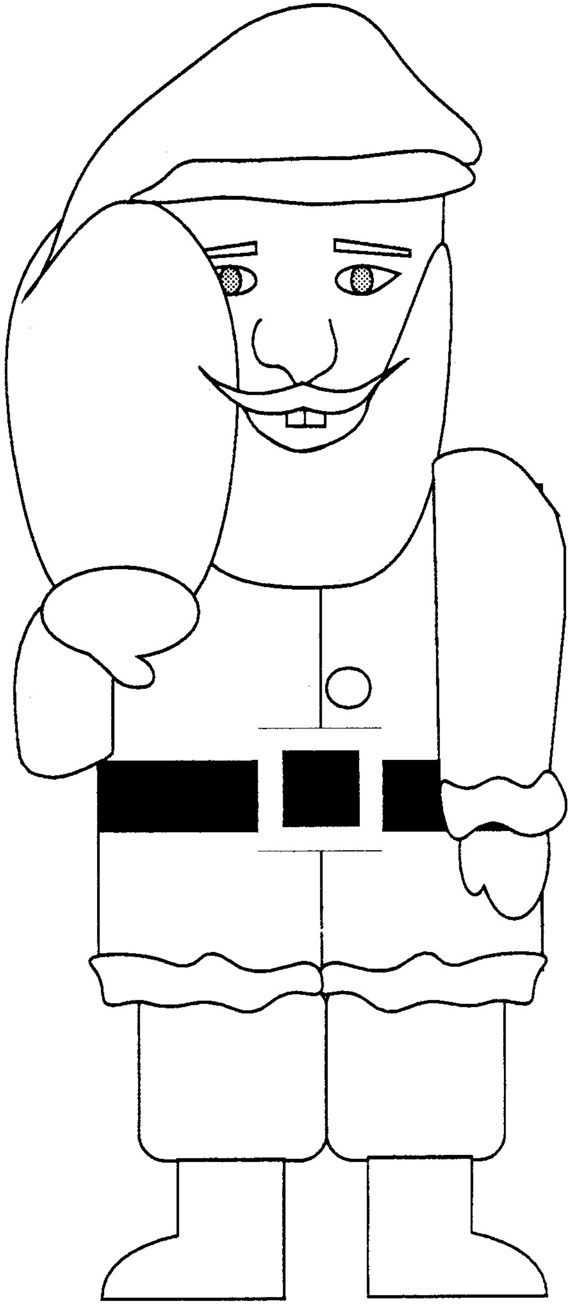

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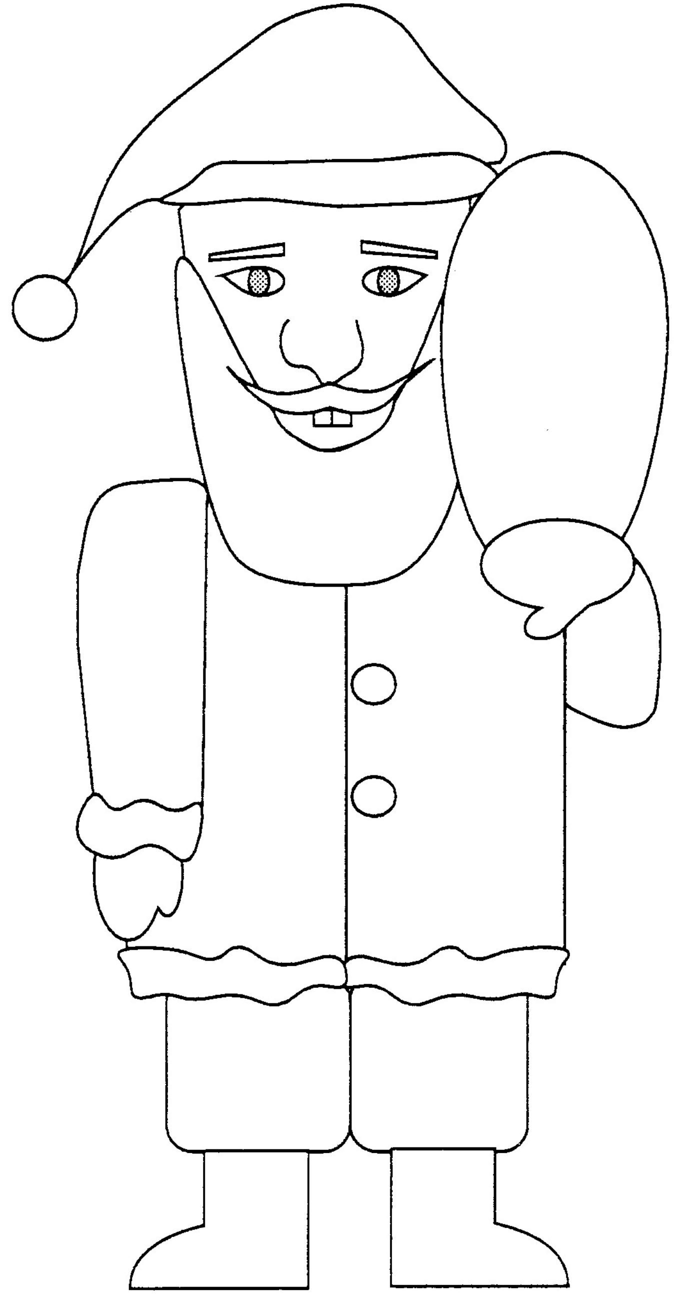

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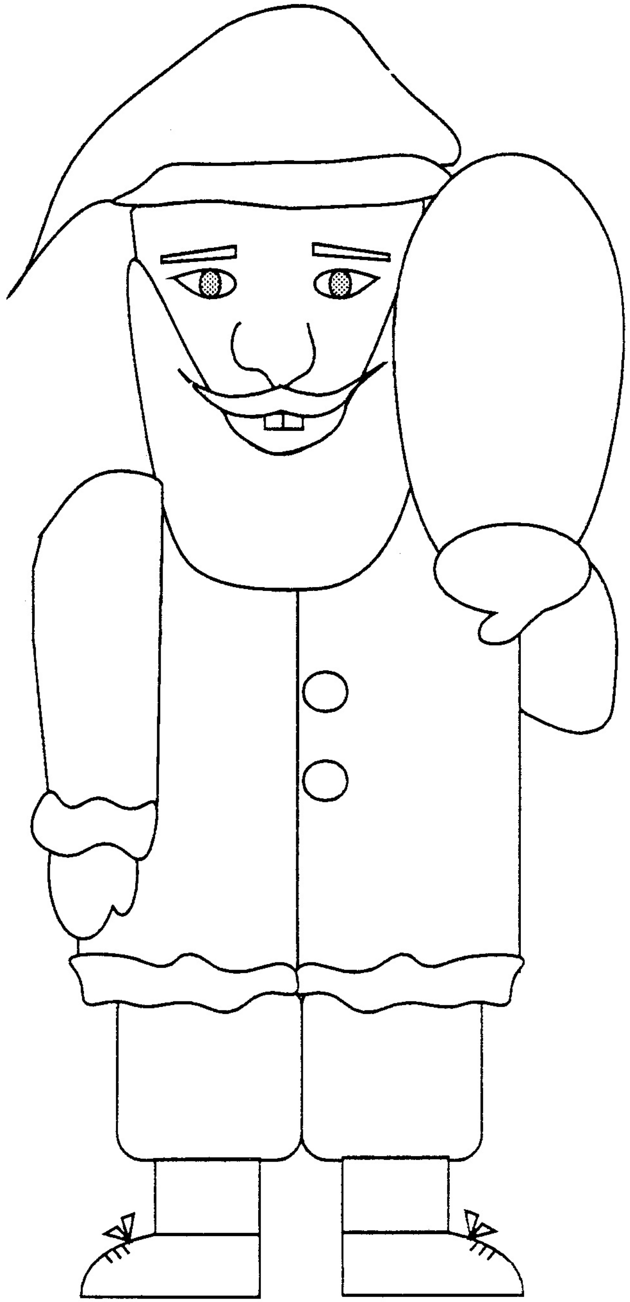

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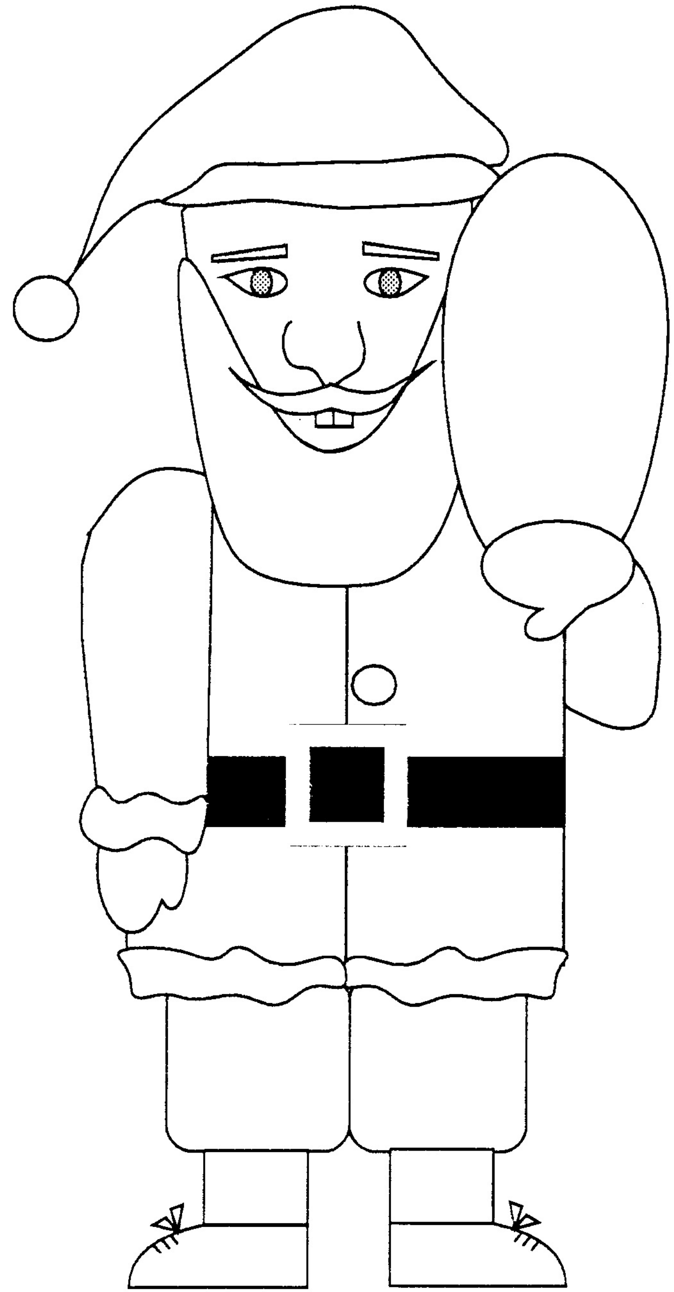

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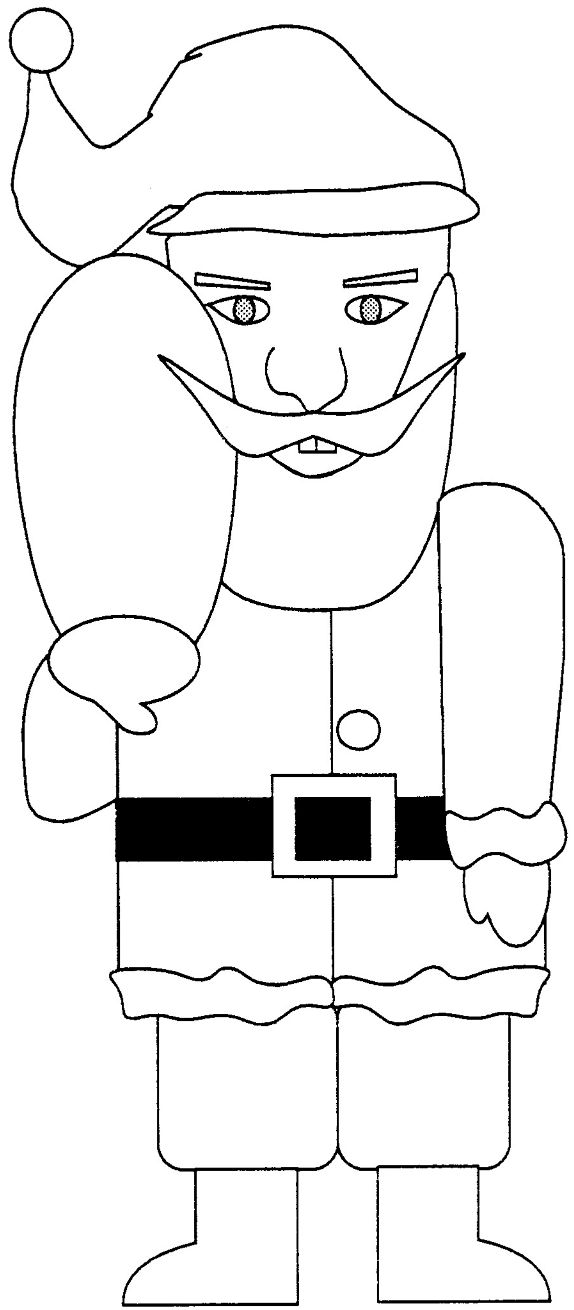

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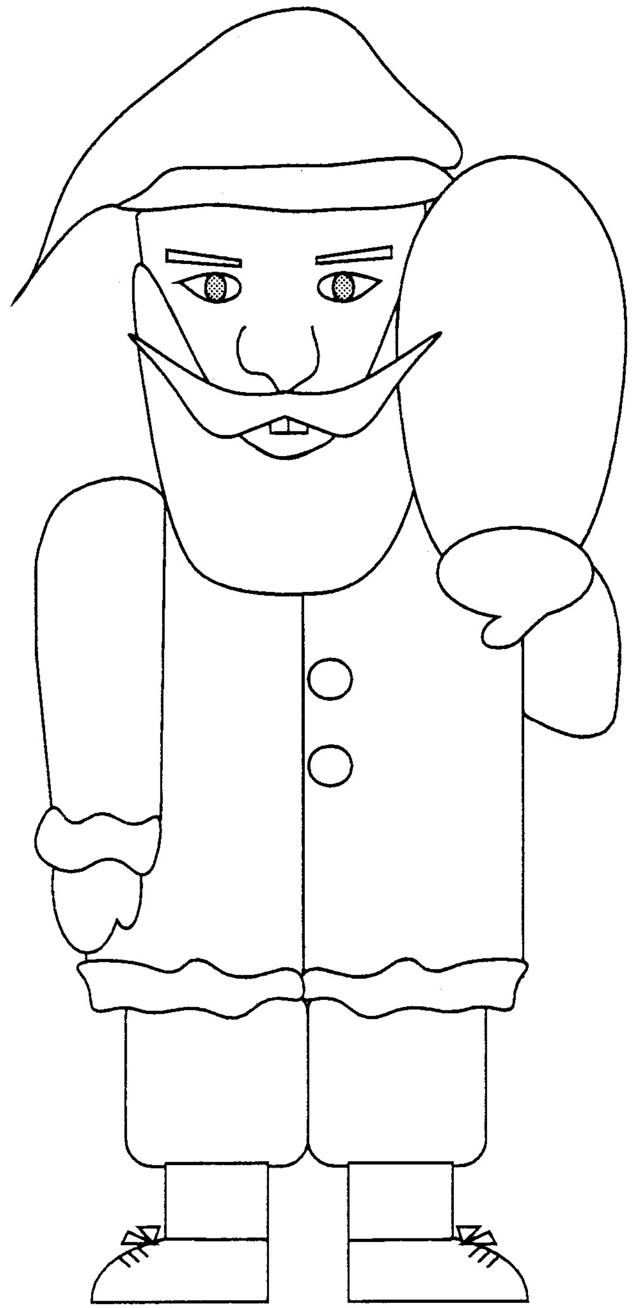

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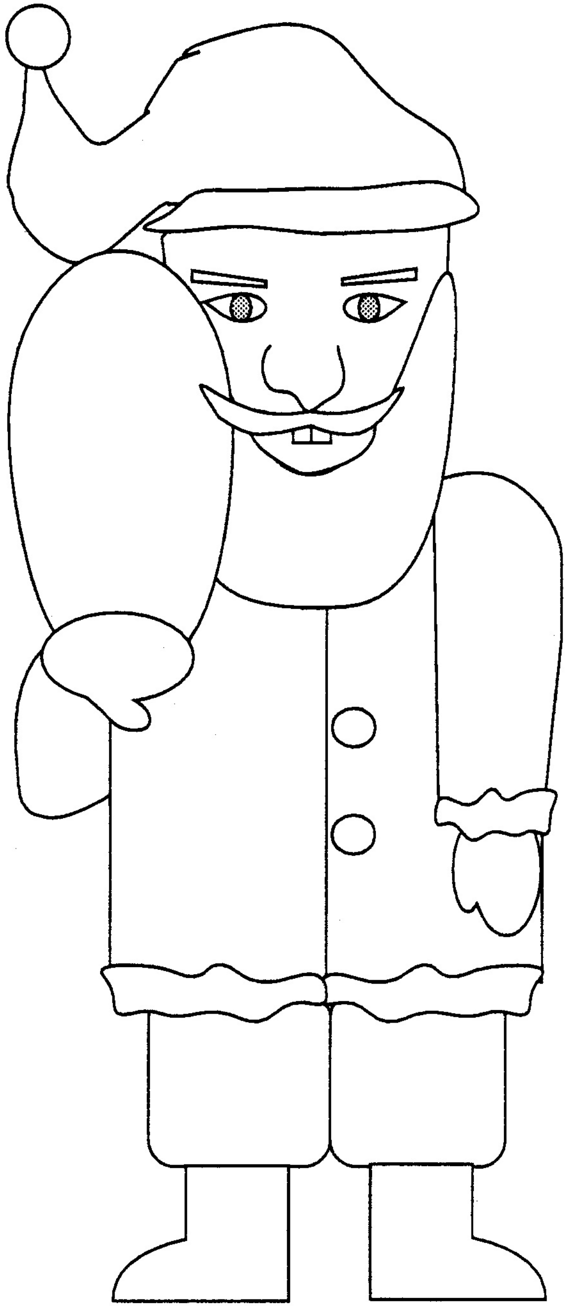

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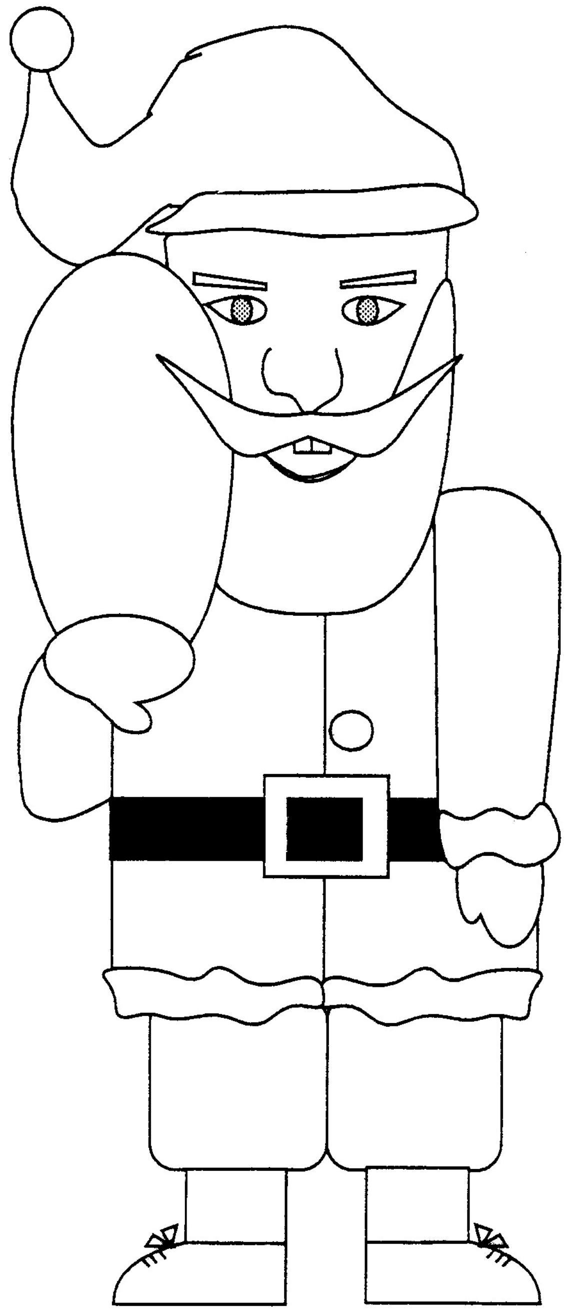

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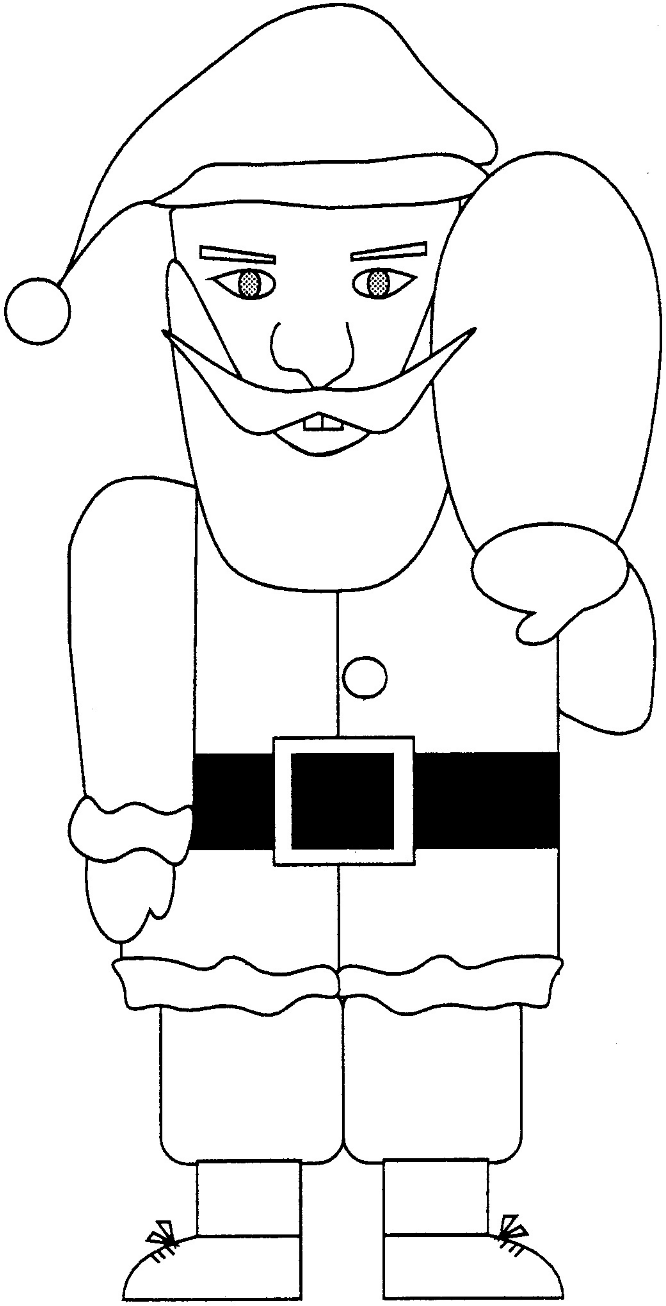

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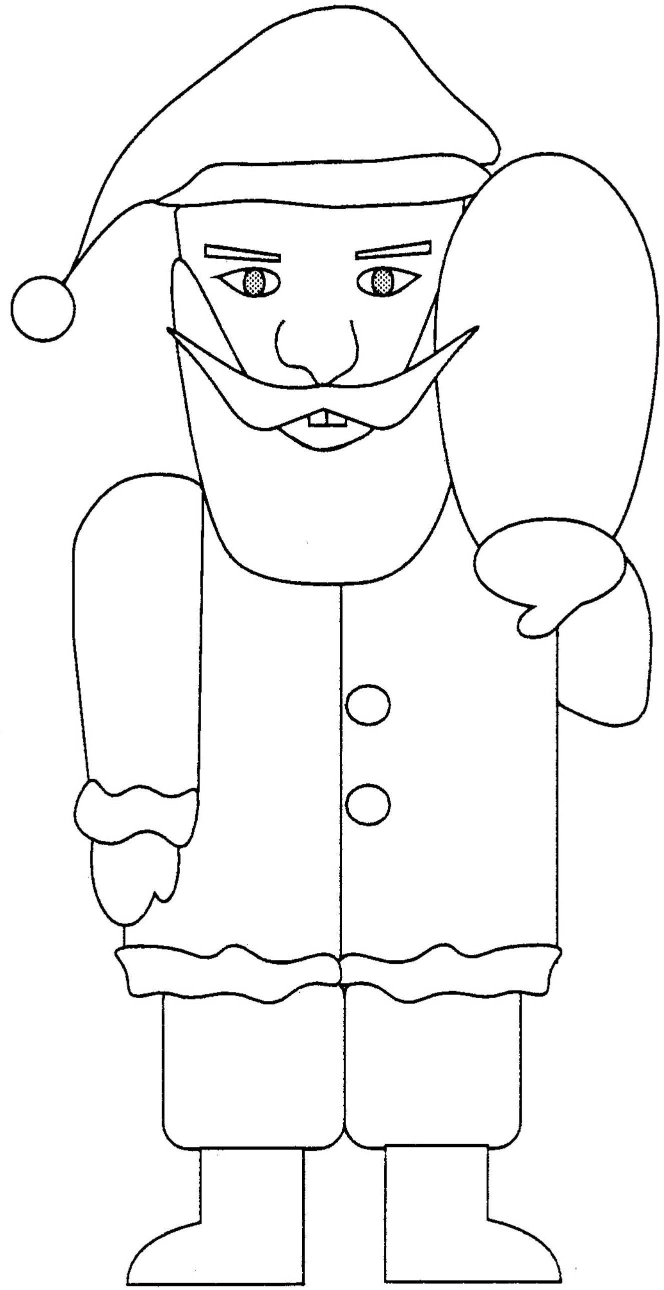

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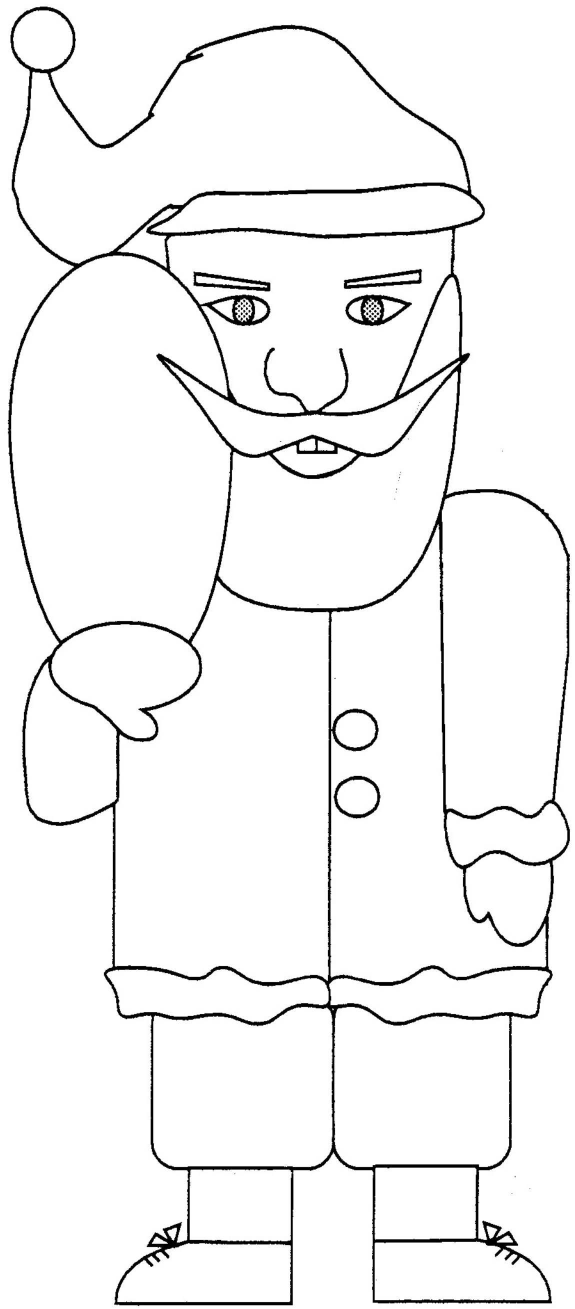

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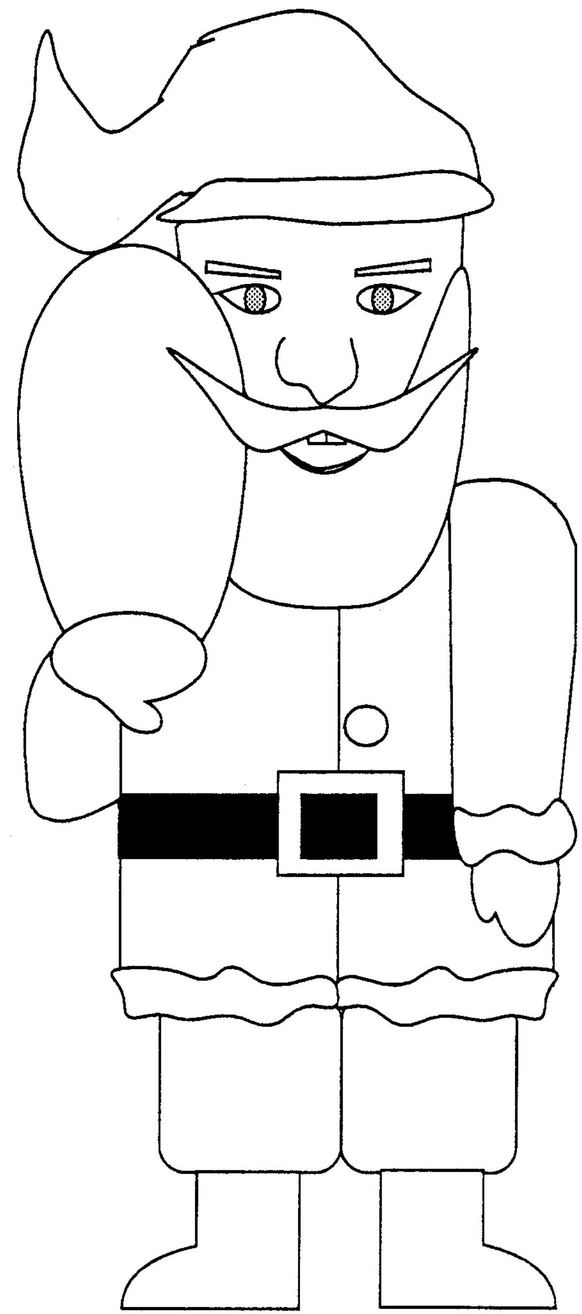

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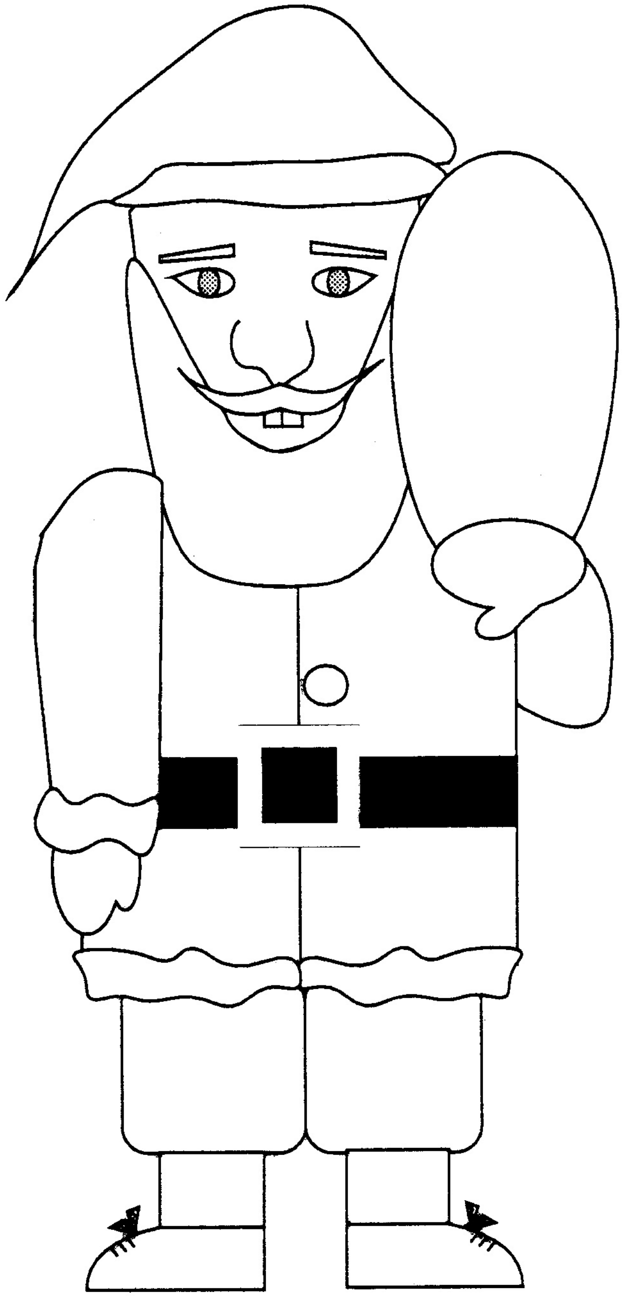

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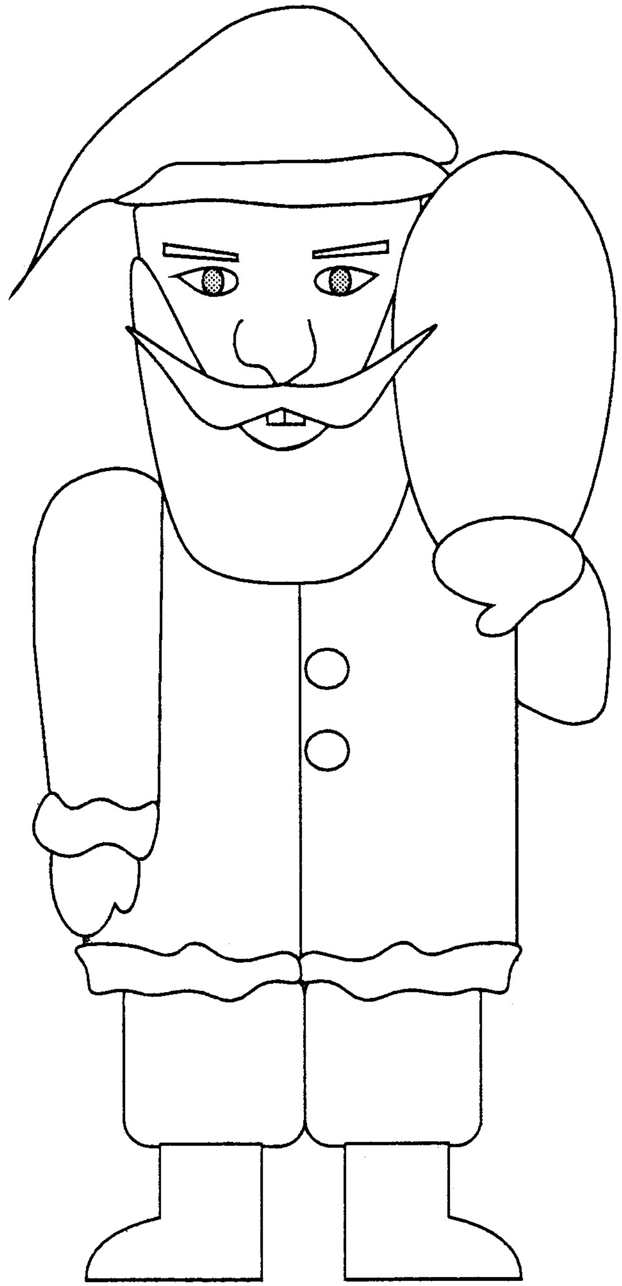

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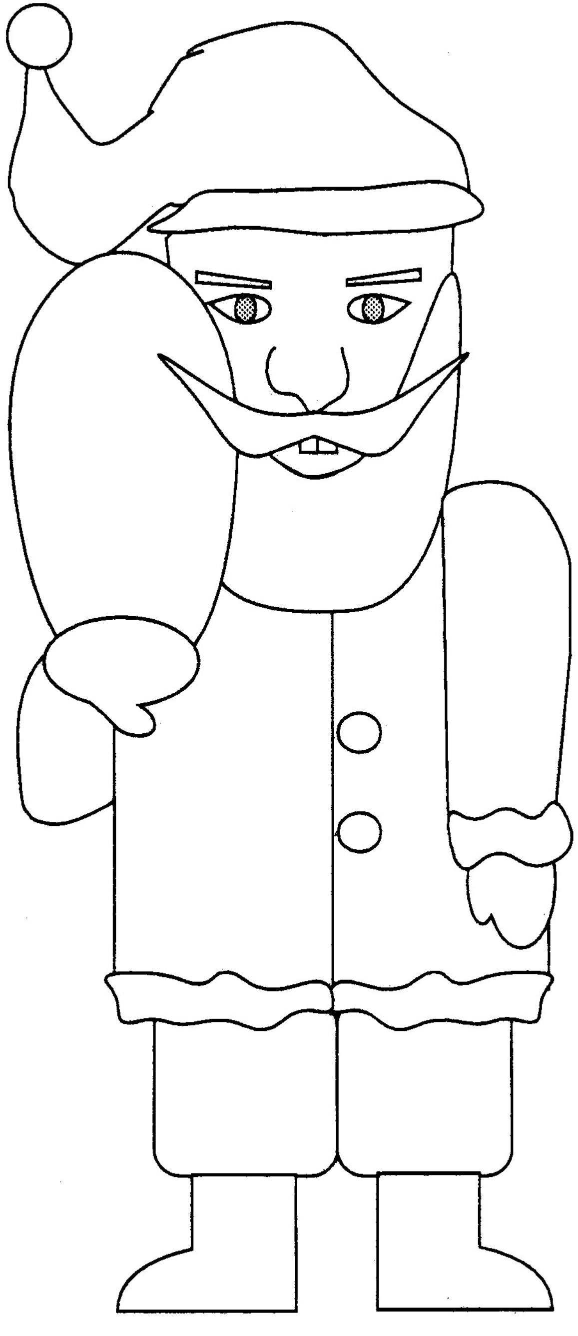

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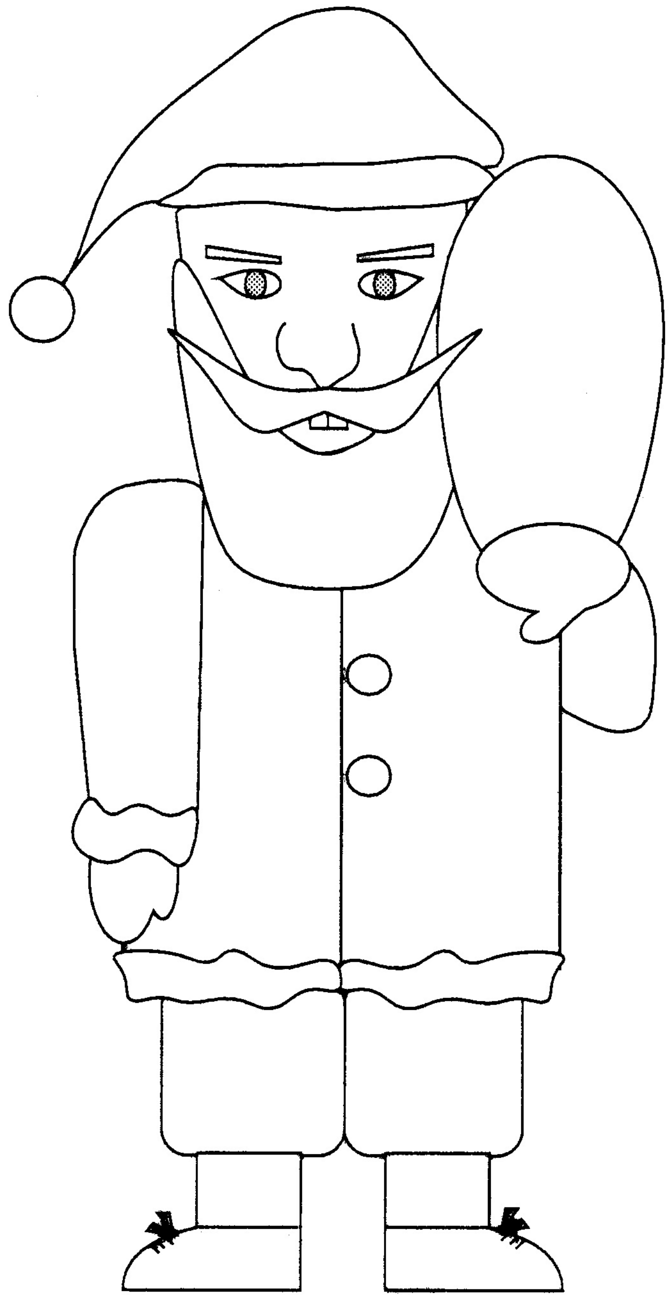

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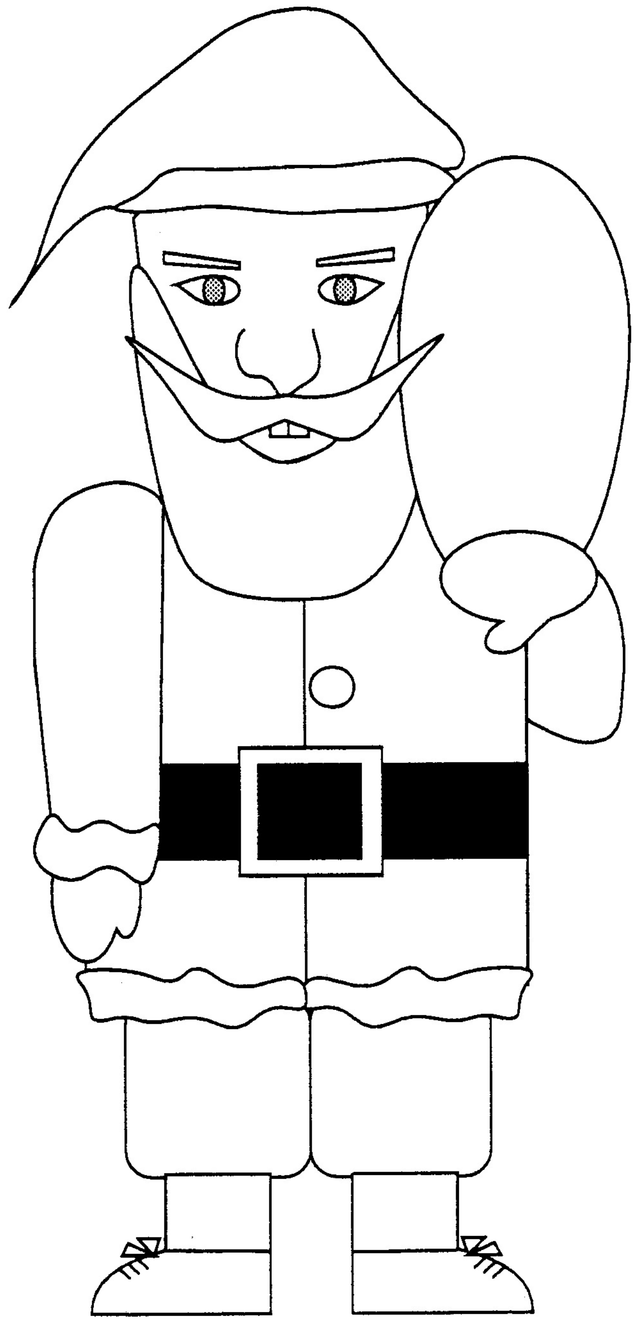

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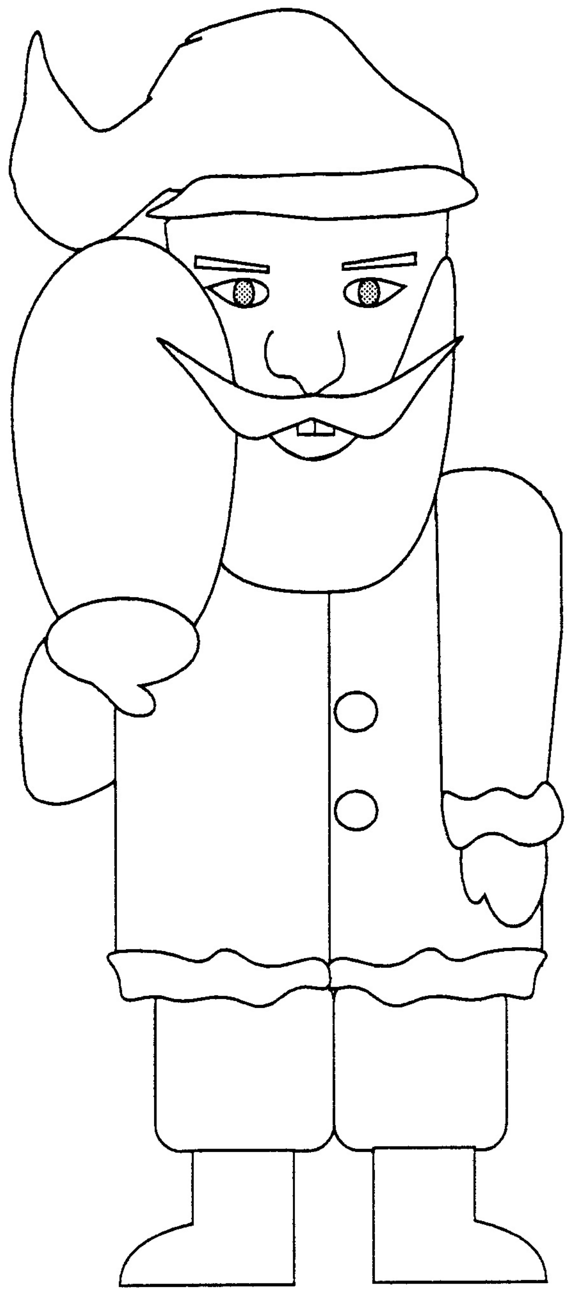

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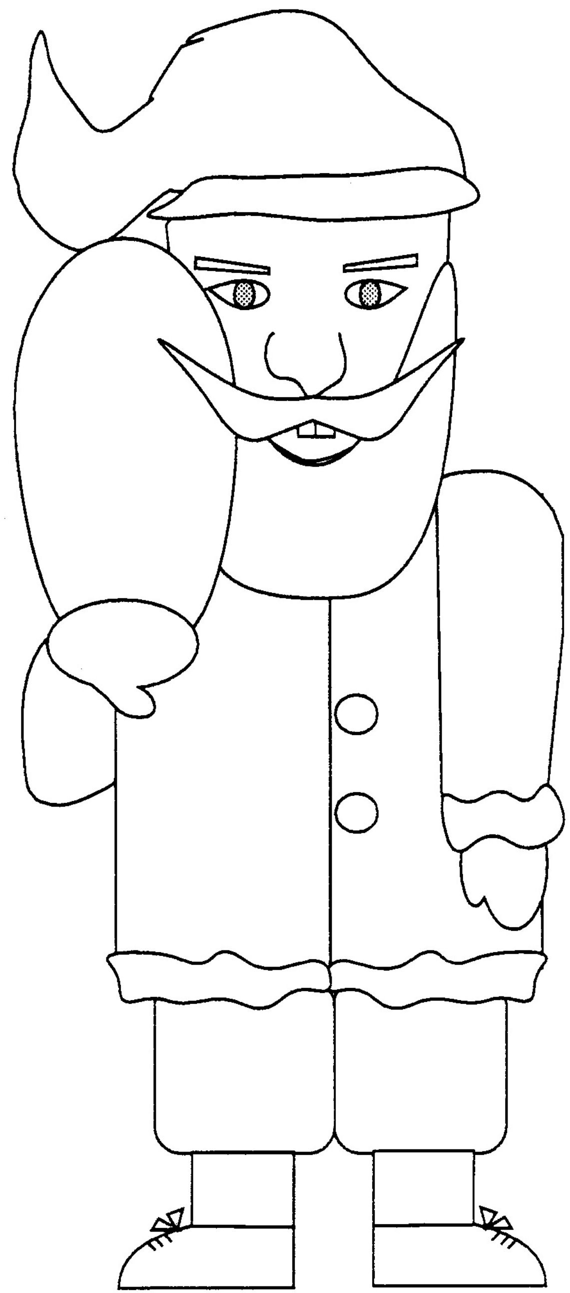

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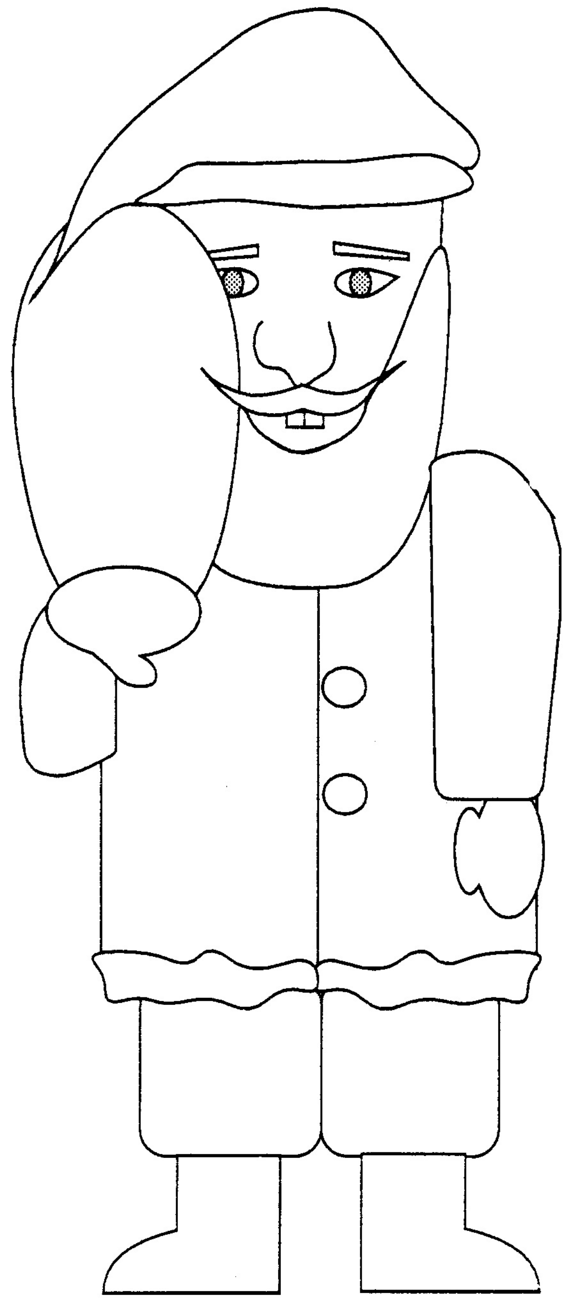

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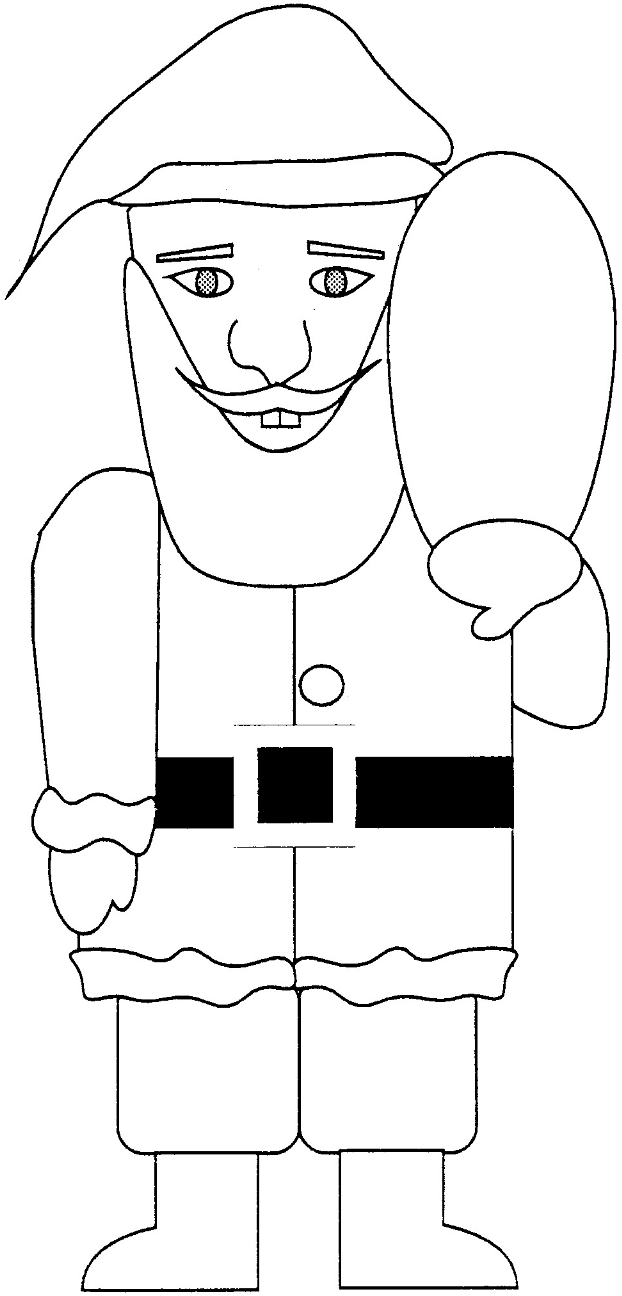

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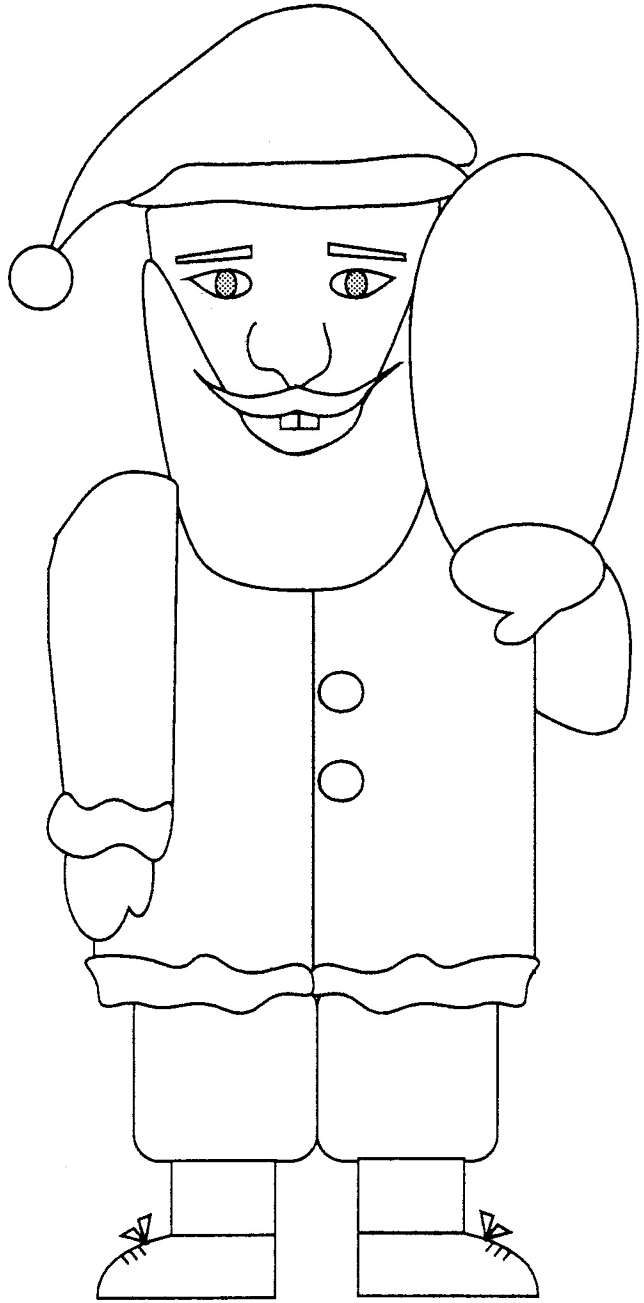


Activity Card 1

Print out a histogram about moustache width then complete these statements:

_____ people have a moustache which is wider than their face.

_____ people have a moustache which is narrower than their face.

Print out a histogram about footwear then complete these statements:

_____ people are wearing boots.

_____ people are wearing shoes.

And one more histogram about buttons then complete these statements.

_____ people have 1 jacket button.

_____ people have 2 jacket buttons.

Activity Card 2

Search for the names of the Father Christmases with a narrow moustache. Name them:

Now, start again and search for those who have a narrow moustache and who are wearing shoes.

Name these:

Activity Card 3

The Reindeer's Friend

Yesterday Rudolph got stuck in the snow. He was rescued by a person who had a lovely narrow moustache. He was wearing stout shoes. His hat got tangled in Rudolph's antlers, but he didn't have a pom-pom. The sun glinted on his shiny belt buckle.

Who might it have been?

Write here: ____________________

Activity Card 4

Who is the Real Father Christmas?

His moustache is wider than his face.

His belt has a big buckle.

His jacket has 1 button.

His hat has a furry pom-pom.

He carries his sack on his right shoulder.

He is wearing long boots.

His name is: ____________________

Find the Father Christmas

Number	Name
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	

Select a picture of a Father Christmas and discover his name through exploring the datafile.

"Who stole the sausages"

This is a different detective scenario which involves a set of 24 dogs. Each dog is represented by a database record with the following field names:

Name:	
Patches:	yes or no
Tailcurl:	up or down
Favfood:	chocolate, lamb chops, biscuits, sausages
Collar:	yes or no
Breed:	Dachshund

Activity card 1 focuses on interpreting a histogram.

Activity cards 2 and 3 suggest simple searches.

Activity cards 4, 5 and 6 outline three different scenarios. Finding the solution involves compound searches.

Suggested extension activities:

1) Creating different scenarios in which clues about the identity of the dog are hidden.

2) Adding further dogs to the file.

3) Adding further fields to each record, for example, the age of the dog, the name of its owner etc.

4) Researching real dogs, finding out about famous dogs, investigating stories about dogs who have performed heroic deeds.

5) Collecting data about pets belonging to the children in the class, or the school, in order to look for some kind of a pattern. For example, are some pets more popular with certain aged children than others, are some pets always popular (why), are some pets very unpopular, what makes someone decide what kind of pet they would like? This could lead to work on the cost of keeping a pet (a spreadsheet might come in useful).

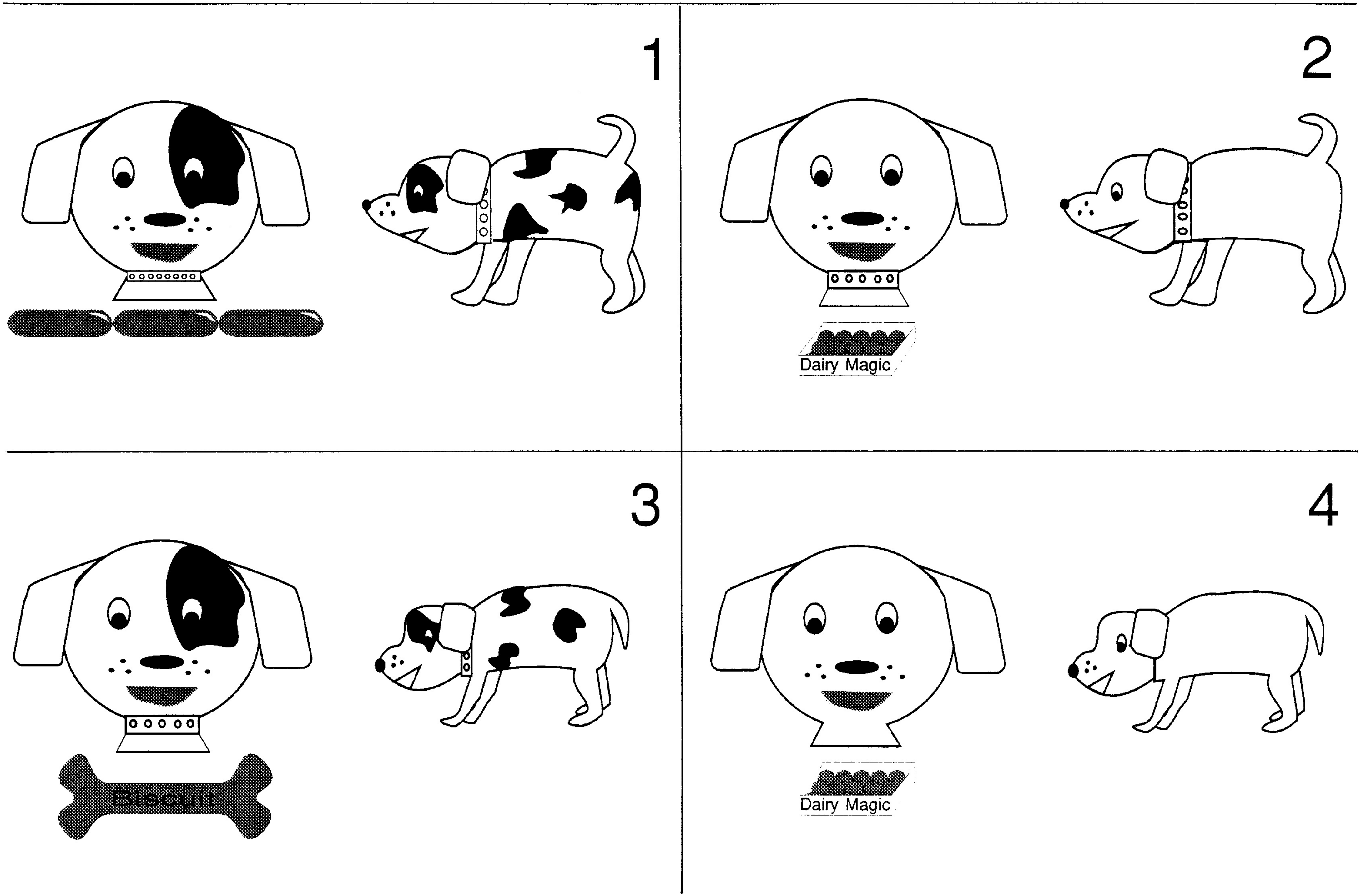
6) Writing from the point of view of a specific animal, in a given situation.

Here is a complete print out of the dogs file:

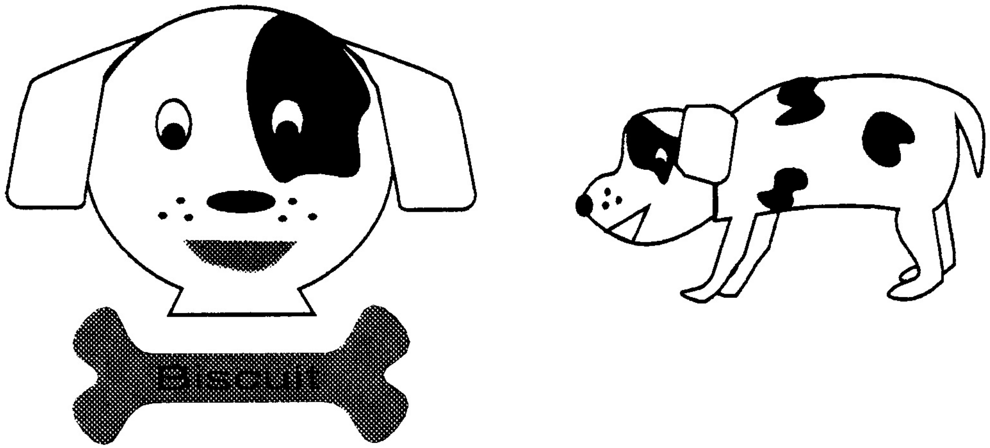
No.	Name	Patches	Tailcurl	Favfood	Collar	Breed
1	Smudge	yes	up	sausages	yes	Dalmatian
2	Wagger	no	up	chocolate	yes	Spaniel
3	Hoppity	yes	down	biscuits	yes	Terrier
4	Bouncer	no	down	chocolate	no	Labrador
5	Loopy	yes	down	biscuits	no	Collie
6	Alex	no	down	chops	no	Dachshund
7	Sleepy	yes	down	chocolate	yes	Cross
8	Bingo	yes	up	biscuits	no	Collie
9	Snoopy	no	up	biscuits	no	Labrador
10	Kipper	yes	down	chocolate	no	Pointer
11	Pluto	no	up	biscuits	yes	Scottie
12	Casper	no	down	sausages	no	German Shepherd
13	Eccles	yes	up	chops	yes	Pointer
14	Dimsie	no	up	sausages	no	Cross
15	Muncher	yes	down	sausages	yes	Spaniel
16	Grumbler	yes	up	chocolate	yes	Collie
17	Mutt	yes	down	chops	yes	Dalmatian
18	Champion	no	up	chops	yes	Scottie
19	Mousy	no	up	sausages	yes	Dachshund
20	Tuppence	yes	up	chocolate	no	Spaniel
21	Shadow	yes	up	sausages	no	Pointer
22	Floppy	no	up	chocolate	no	Terrier
23	Casey	no	down	chops	yes	Cross
24	Gnasher	yes	up	biscuits	yes	Terrier

An example record from the dogs file.

Number	10
Name	Kipper
Patches	yes
Tailcurl	down
Favfood	chocolate
Collar	no
Breed	Pointer


1
2
Dairy Magic
3
Biscuit
4
Dairy Magic

5



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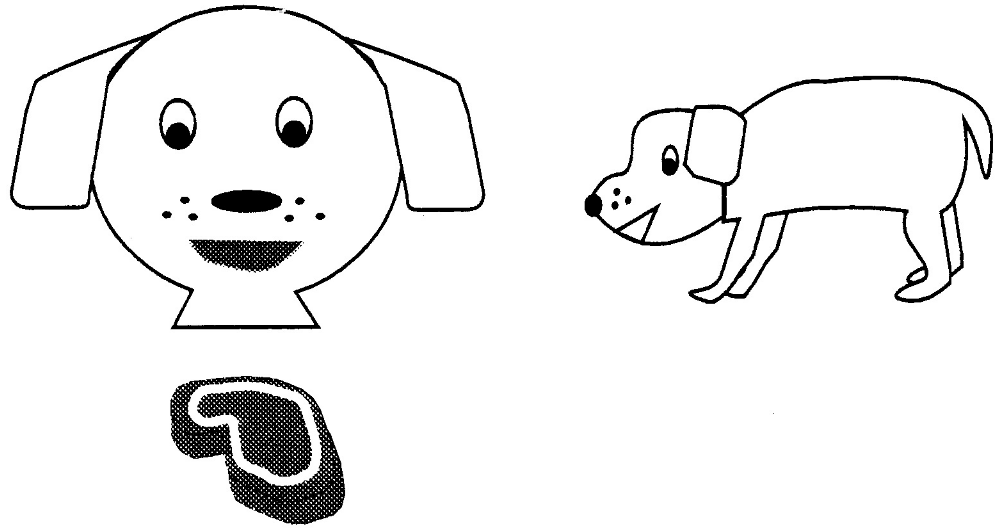

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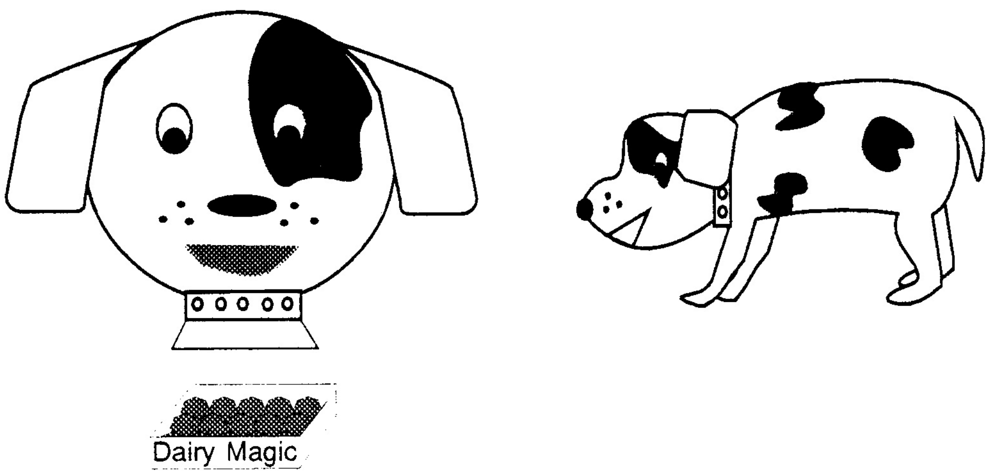

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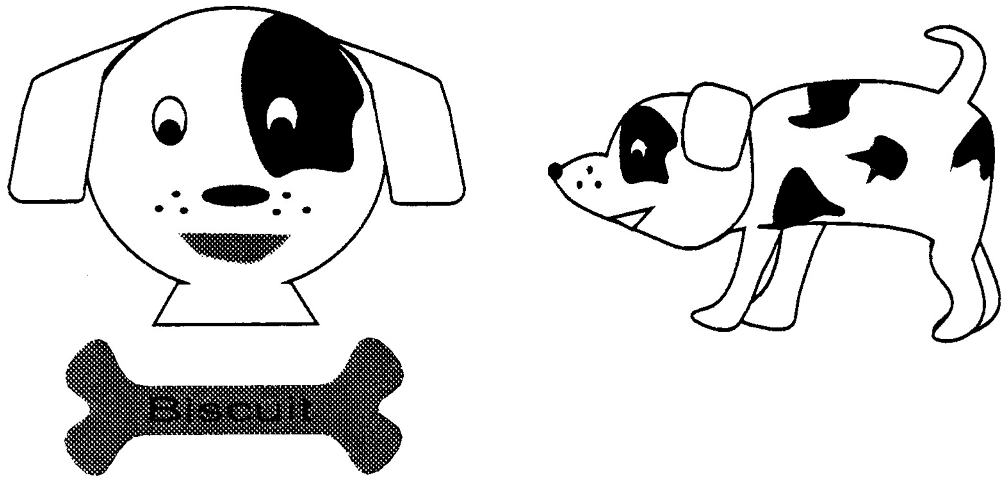

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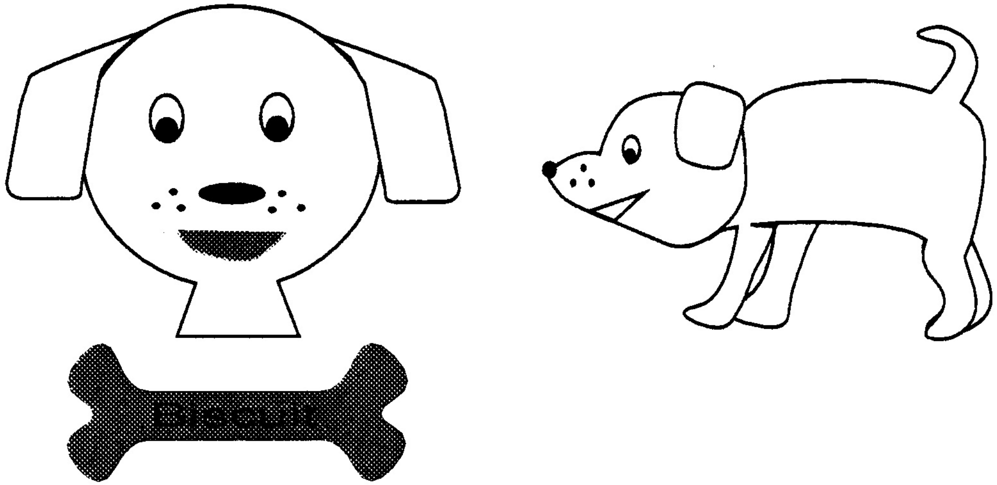

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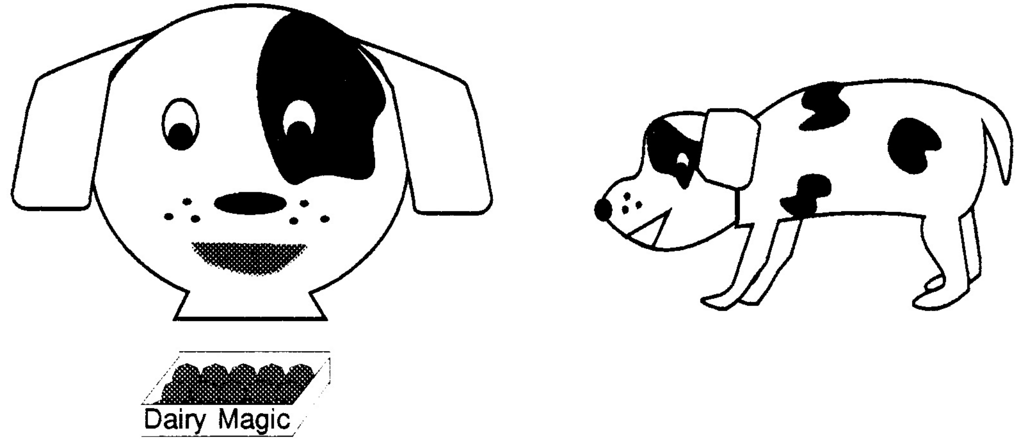

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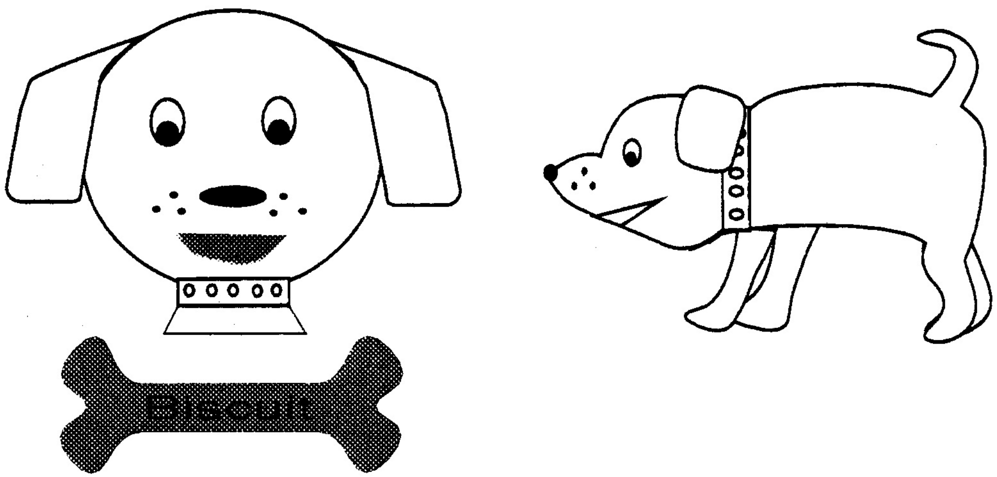

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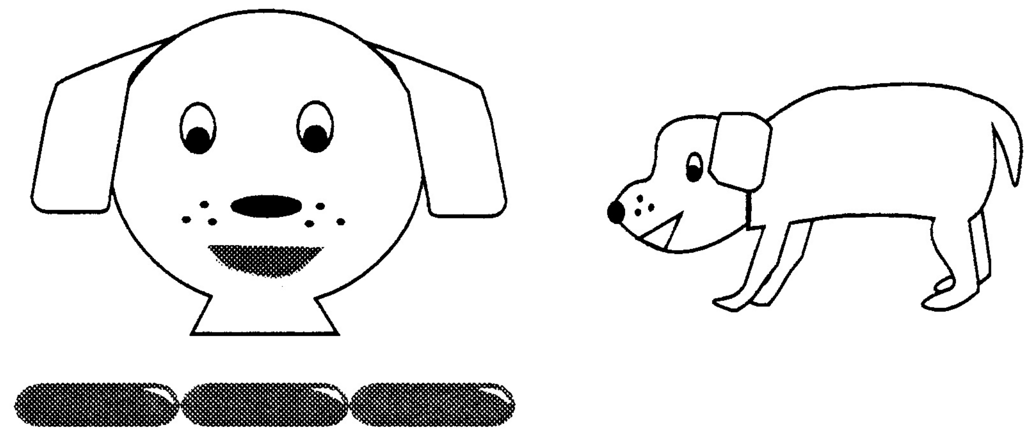

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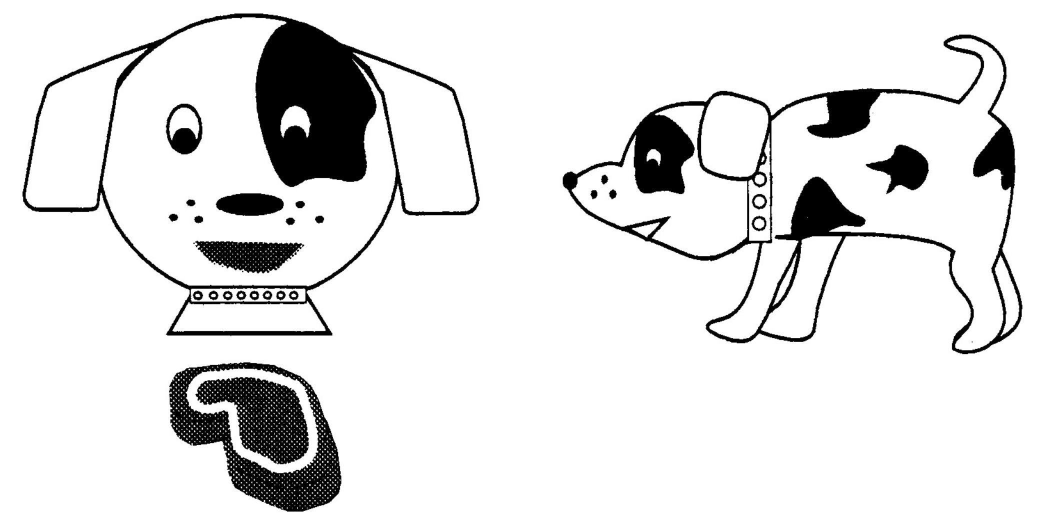

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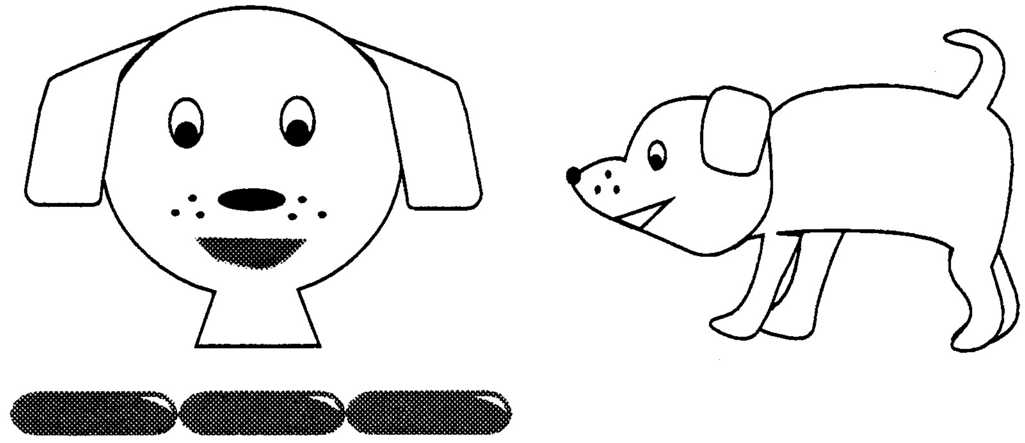

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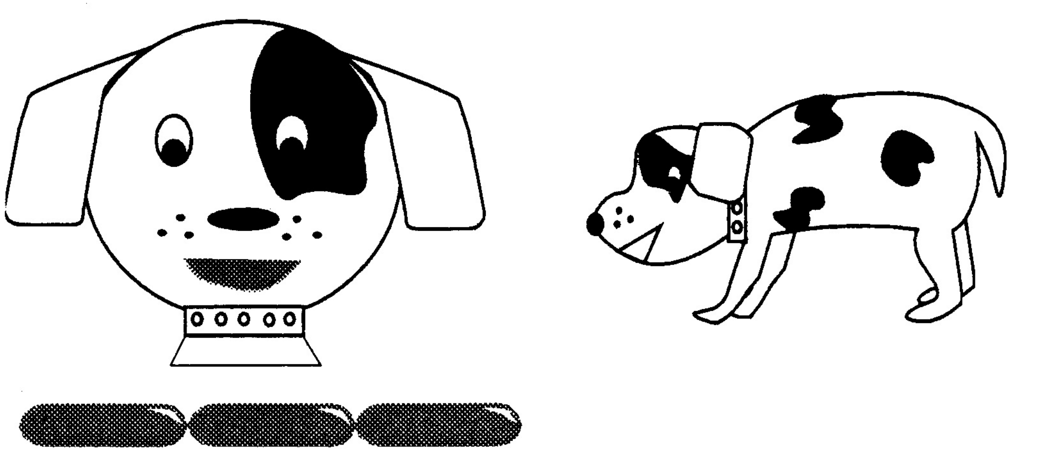

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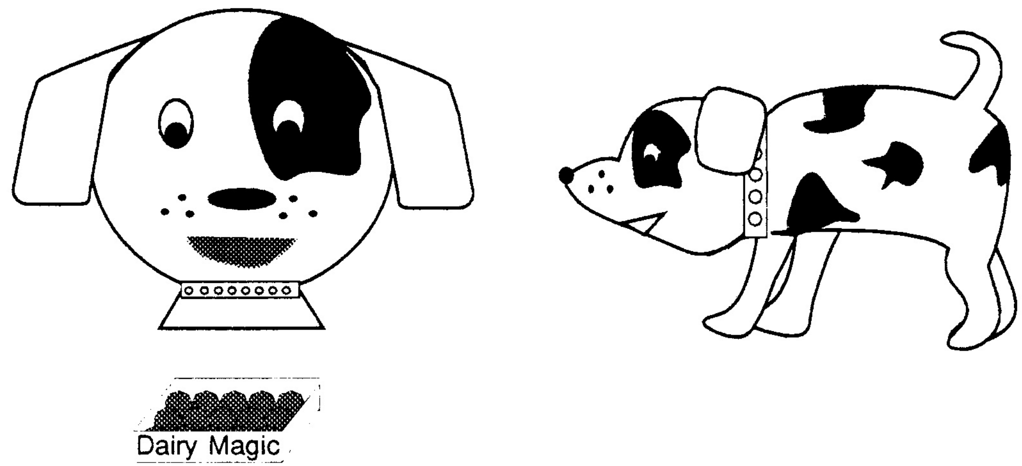

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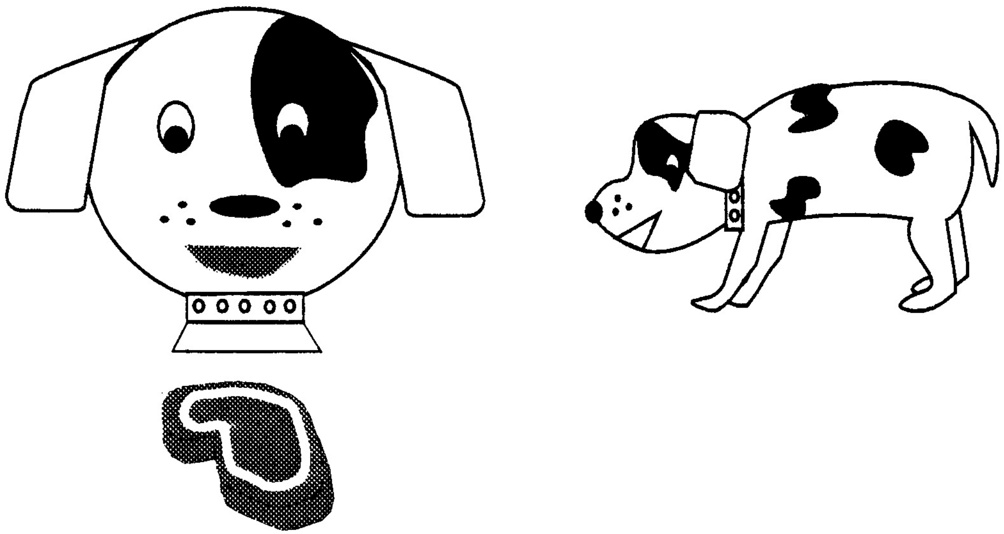

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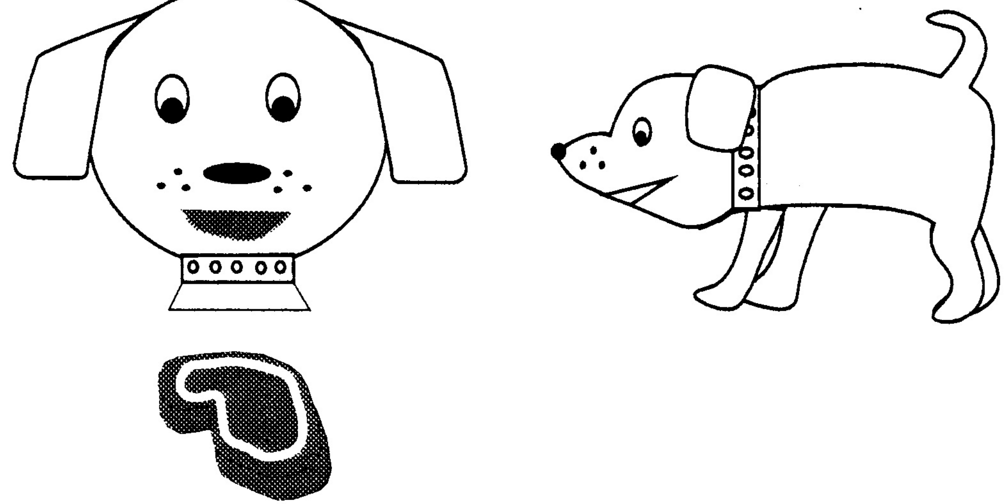

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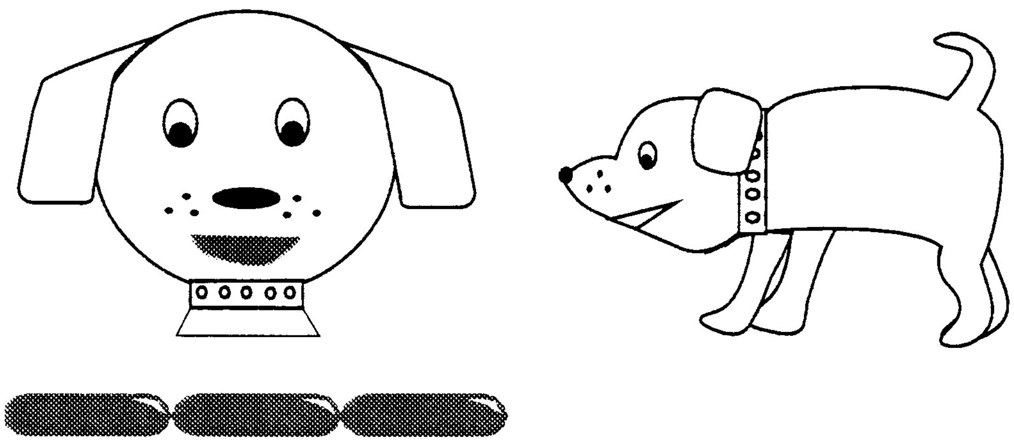

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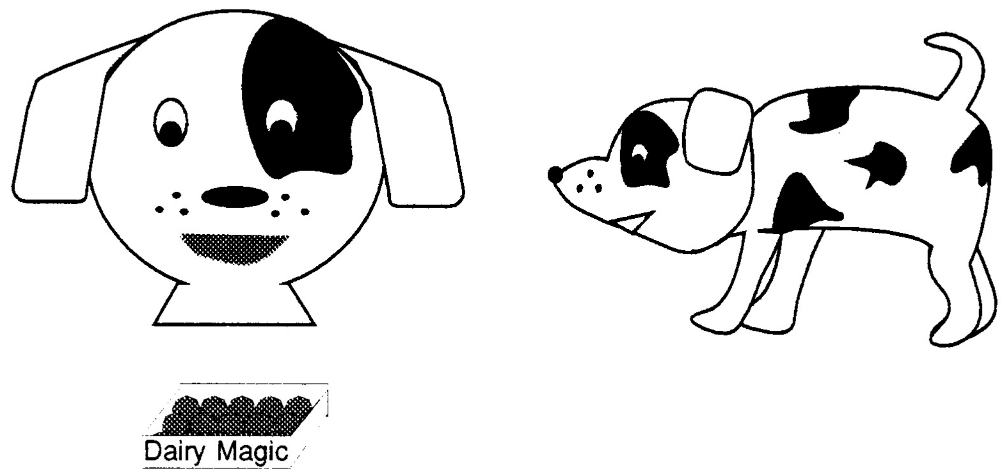

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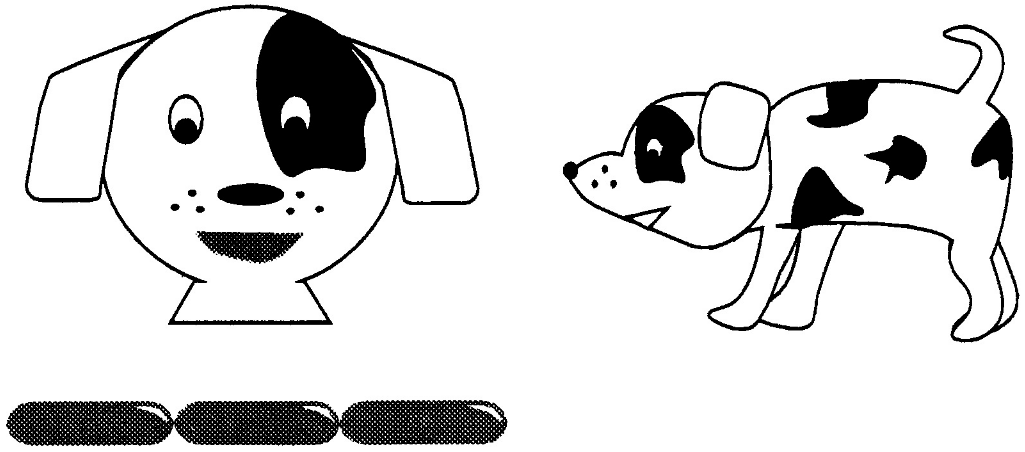

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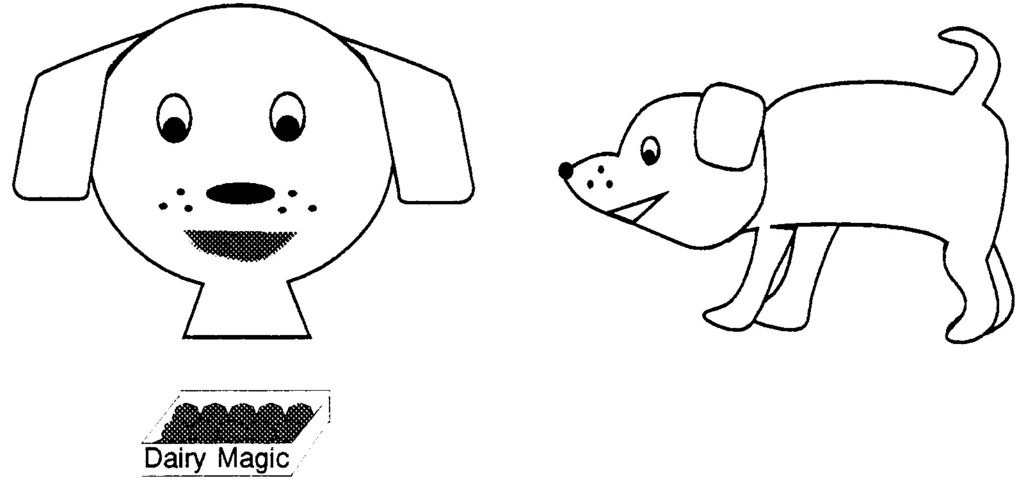

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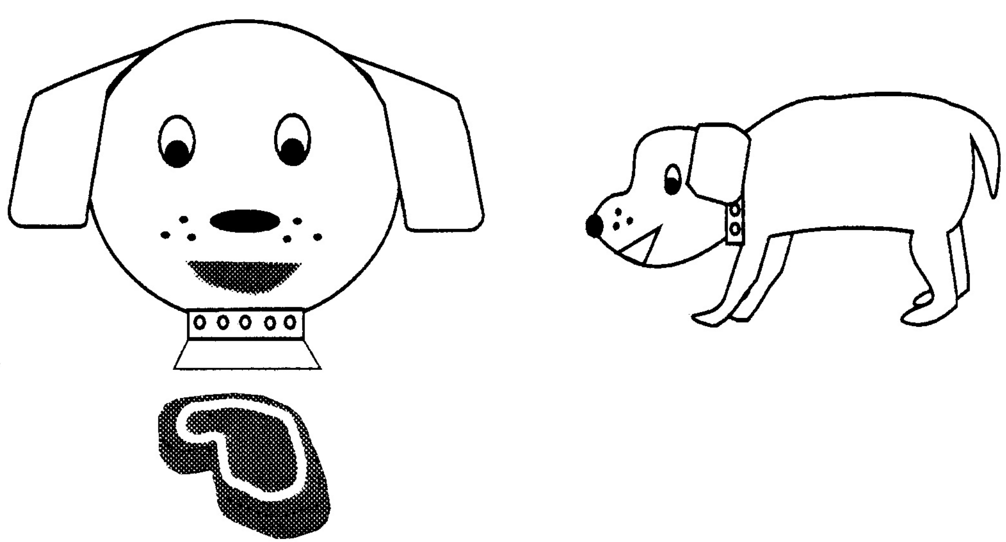

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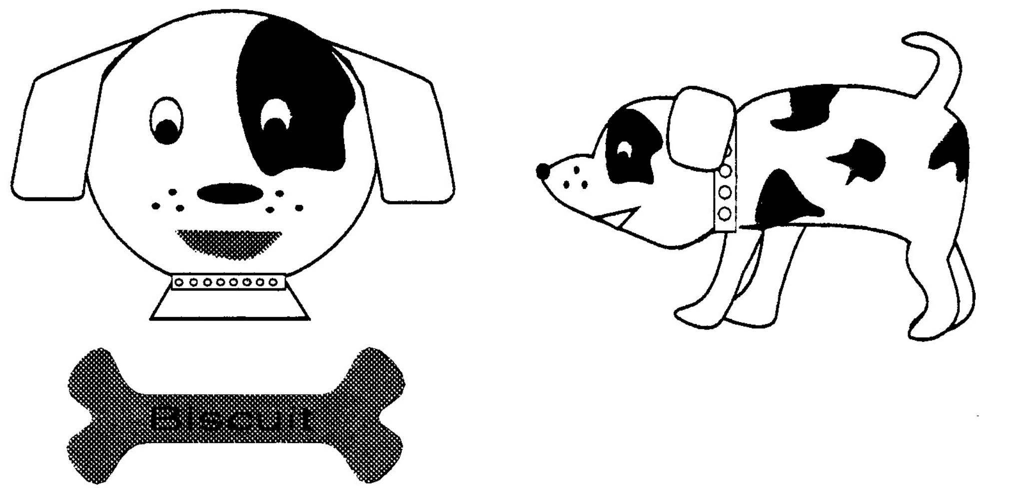

Activity Card 1

Print out a histogram of the dogs' favourite foods.

What does it show?

_____ dogs like chocolate.

_____ dogs like biscuits.

_____ dogs like sausages.

_____ dogs like chops.

Out of the dogs in the file, the most popular food is: _______________.

The least popular is: _______________.

Activity Card 2

Search through the data and find the names of all the dogs who like chocolate.

Write them here: ____________________

____________________ ____________________

____________________ ____________________

____________________ ____________________

Search the chocolate lovers and find those dogs who have a tail which curls up.

Write their names here:

____________________ ____________________

____________________ ____________________

Activity Card 3

Search through the data and find the names of the dogs who are cross breeds.

Write their names here:

________________ ________________

____

Which of these dogs has a tail which curls down?

________________ ________________

Which of these dogs is a cross breed, has a tail that curls down and likes chocolate?

____

Activity Card 4

The Sausage Stealer

Some sausages were stolen from Jake's Butchers on Saturday. Jake has been questioned and the results of this questioning are as follows:

The suspect:

is a dog.

does not have any patches.

may have been a Dachshund but the eye witness is not certain of this fact.

wears a collar.

What is the name of the sausage thief?

____

Activity Card 5

The Brave Rescuer

Last night a fire broke out in Mary Jones' house. Her daughter, Susan, aged 6, was rescued by a dog who led her out of the smoke filled house.

Susan said "He had a wet nose and his tail brushed down against my toes. He whined at me until I held onto his collar. Then he led me out. It was difficult to see clearly but I think he was the same colour all over. When we were outside the fire fighter gave him a chocolate, but he spat it out."

What is the name of the rescuer?

____

Activity Card 6

Digger

Mrs Phyllis Law discovered a huge hole in her flower bed this morning. At the bottom was a bone shaped a bit like a rib. Her neighbour, Michael Grove, aged 57, said he'd heard a scraping noise in the night and when he'd looked out of his window he'd seen a large dog, with light patches and a long tail which almost touched the ground, digging the hole. He had shouted and the dog had run away with his collar glinting in the moonlight.

Which dog was it? ________________

Name the dog

Number	Name
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
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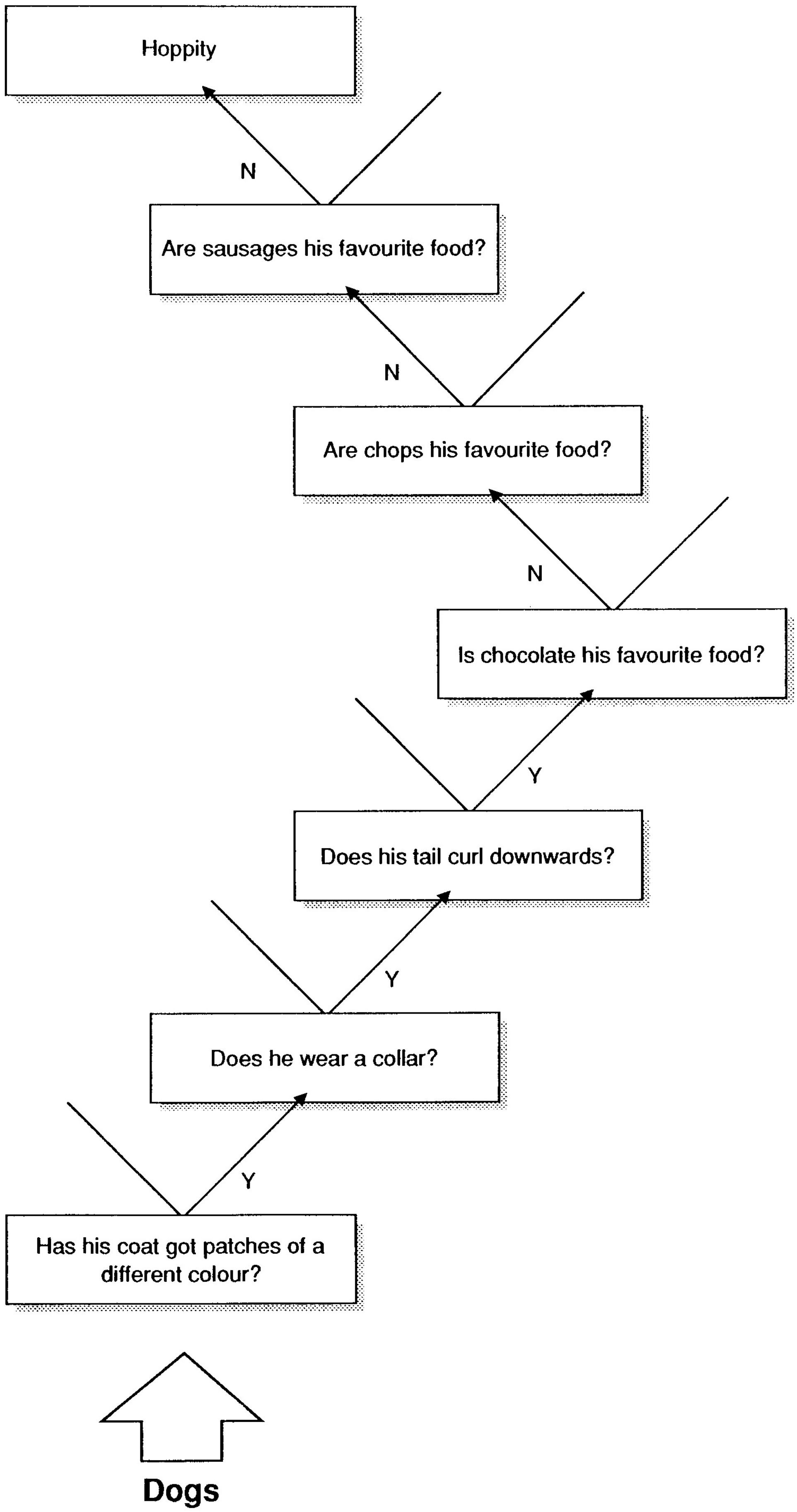
Select a picture of a dog and discover his name through exploring the datafile.



Example of a tree structure constructed using Sorting Game

Puzzling Shapes

This shape contains at least one right angle
It has 1 line of bilateral symmetry
It will not tessellate
It is not made up entirely of vertical and horizontal lines
It is concave
It has rotational symmetry of order 1
It has 9 sides

What is it?

Young children operate an information retrieval system in their heads (we hope) during most of their activities at school. They start to learn to recognise shapes, they learn what to look for, they can describe the difference between a square and a rectangle. They can learn to draw a variety of shapes with increasing degrees of accuracy. They begin to apply different tests, they look for lines of symmetry (and can describe different types of reflection). They make tessellating patterns or pictures. They gradually build up their own database about shapes. 'Puzzling Shapes' is designed for exploration by those pupils who know how to:

a) use a data handling program;
b) recognise, name, describe and sort 2-D shapes;
c) recognise reflectional and rotational symmetry;
d) formulate hypotheses.

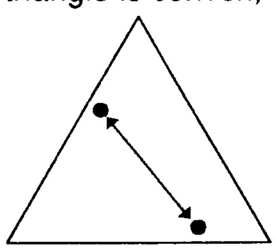
'Puzzling Shapes' consists of a database which refers to 48 2-D shapes, together with cards illustrating every shape, and some ideas for investigations. Each shape has been given an imaginary name on the database. This ensures pupils have to search the database by examining the attributes of the shape - they can't bypass the process by simply searching for the conventional name of the shape.

The information describing each shape has been stored under the following headings:

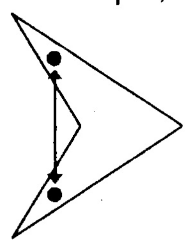
Field name	Notes	Example
Name	Imaginary	Spinner
Sides	Number of sides	12
Bilateral	Number of lines of bilateral symmetry	0
Rotational	Degree of rotational symmetry	3
RightAngle	Has the shape got at least 1 right angle?	yes
Vert/Horiz	Is every line either vertical or horizontal?	no
Outline	Is the shape convex or concave?	concave
Tessellate	Will the shape tessellate perfectly?	no

To test if a shape is convex or concave:

Select any two points inside the shape and join them up with a straight line. If this line (and any other line you could draw) lies entirely within the shape, then it is convex. If part of the line passes outside the shape then it is concave. Hence a triangle is convex, where an arrow head, for example, is concave.



Convex



Concave

Using the database:

As stated earlier, this activity is designed for pupils who are familiar with GRASS and with information handling activites. Nevertheless, they might like to start by conducting simple searches of the data, or by sorting the shapes according to criteria they have defined so that they get a 'feel' for the data.

Starting points could be:

* drawing a histogram of the number of sides that each shape has;
* searching for shapes which have 3 sides;
* sorting the shapes according to the number of sides of each shape;
* creating a histogram of the order of rotational symmetry for each shape;
* creating a histogram of the lines of bilateral symmetry in each shape;
* searching for shapes which have rotational symmetry of order 4;
* sorting the shapes on the degree of rotational symmetry;
* searching for shapes which have three axes of bilateral symmetry;
* sorting the shapes according to the lines of bilateral symmetry;
* searching for shapes which are convex, or shapes which are concave;
* searching for shapes which have at least one right angle;
* searching for shapes which tessellate perfectly (ie. floor tiles of this shape should be able to cover the whole of a floor without the need for a second set of tiles of a different shape).

Naming shapes

While the above activities are useful for teaching pupils how to find information in a database, they do not necessarily promote mathematical investigation - to do this the pupils will need to identify the properties of different shapes in order to match them to their correct names.

The shapes on the cards can be used in various ways. The main activity involves the pupils examining the properties of a particular shape and searching the database for its appropriate name (this is similar to the kind of activity which involves identifying a burglar, or a pirate, from a descriptive scenario). The outlines of individual shapes could be incorporated into displays in the classroom with a blank label which would be completed once the shape had been named (discovered!). The search is likely to be complex but it can be built up in stages. For example, if the shape is a four pointed star the first search criteria might be

	Sides	is equal to	8

this would be refined by adding the following:

AND	Outline	is the same as	Concave

and refined again by adding:

AND	Rotational	is equal to	4

and one shape entitled 'Badge' will be found.

A sheet on which the results of searches can be recorded is included as part of the resource pack. The columns on the sheet are not intended to be completed in full - pupils simply record in the appropriate columns the information that was used for the purpose of searching. This also enables the teacher to check on what the pupils have done.

Once the pupils are familiar with the data, and have identified the shapes as outlined above, they can start to look for relationships between different sets of data. They can produce a hypothesis to use as a starting point and then they can explore possible connections. The questions that follow are open ended and exploring potential solutions can lead to interesting mathematical discussions. Some of the ideas below cannot be answered from the database alone - pupils will need to examine the cards with the shapes on. This is because the database may not hold all the information they require. For example it does not give information about whether all the sides of a shape are the same length, or whether a shape contains reflex angles or whether lines of bilateral symmetry are horizontal.

The Puzzling Shapes Database

An example of a typical record:

Name	Abbey
Sides	6 sides
Bilateral	0 lines
Rotational	1 order
Right Angle	yes
Vert/Horiz	no
Outline	concave
Tessellate	no

Definition of Field Names

Name

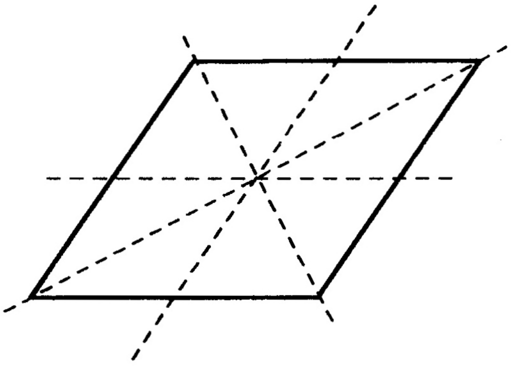
This field holds the imaginary name of the shape. You need to search the database to find what name has been given to the 'square'. To do this you need to think about the properties that the square possesses.

Number of sides

This field holds the number of sides each shape has. You just need to count them. Only shapes with straight sides have been included in the database.

Bilateral

This field holds the number of lines of bilateral symmetry that each shape has. Bilateral symmetry occurs if you draw a line across the shape and the shape on one side of the line is a mirror image of that on the other side. A mirror is a useful tool to check lines of bilateral symmetry. Put it along each of the four lines in the diagram below and decide which of them is a line of bilateral symmetry.



Rotational

This field holds the degree of rotational symmetry that a shape has. Imagine that the shape has been put onto a block of wood and its outline drawn and then cut out so that the shape will drop through. Without turning the shape over, but only rotating it, work out how many ways you could 'post' the shape through the hole. In the case of the square the answer is 4.

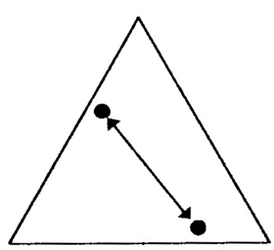
Right Angle
This field holds data about whether or not the shape has at least one right angle (the answer is recorded as either 'yes' or 'no').

Vertical and Horizontal lines
This field holds either 'yes', if every line in the shape is either vertical or horizontal, or 'no' if the shape has at least one sloping line.

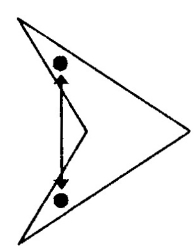
Outline
This field holds either 'concave' or 'convex'.

To test if a shape is convex or concave:

Select any two points inside the shape and join them up with a straight line. If this line (and any other line you could draw) lies entirely within the shape, then it is convex. If part of the line passes outside the shape then it is concave. Hence a triangle is convex, where an arrow head, for example, is concave.



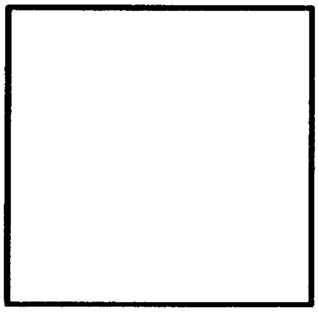
Convex



Concave

Tessellate
This field holds data about whether or not the shape will tessellate perfectly. A shape will tessellate if you could tile a floor with tiles of that shape only and leave no gaps. To test for tessellation, make a copy of the shape on card, and then, using it as a template, draw round it several times. If the paper can be covered without leaving any gaps between each shape then it will tessellate.

1



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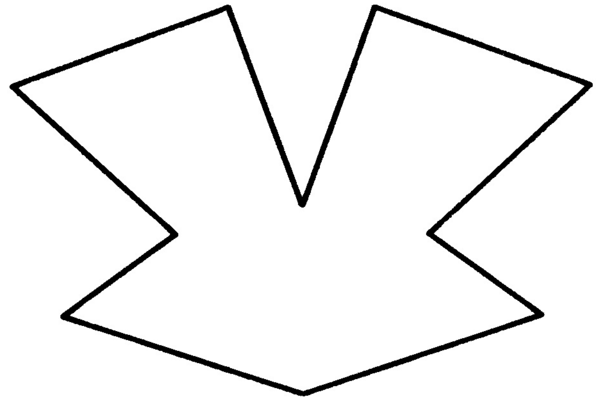

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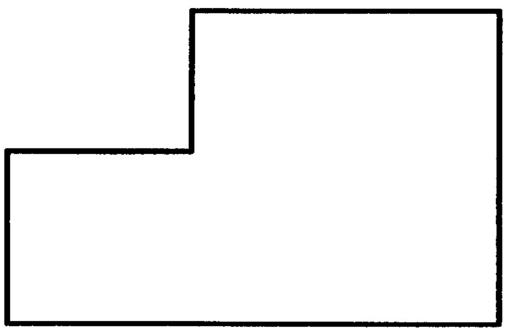

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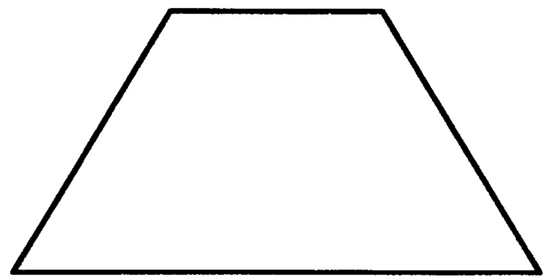

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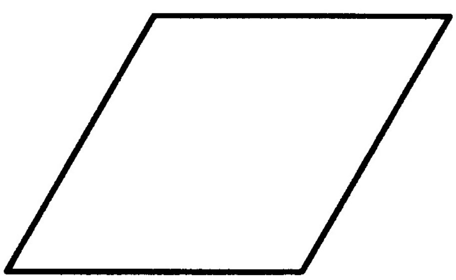

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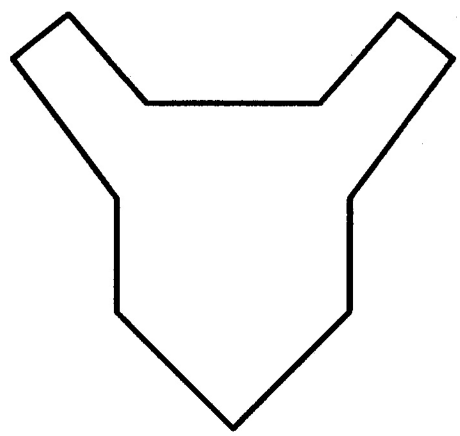

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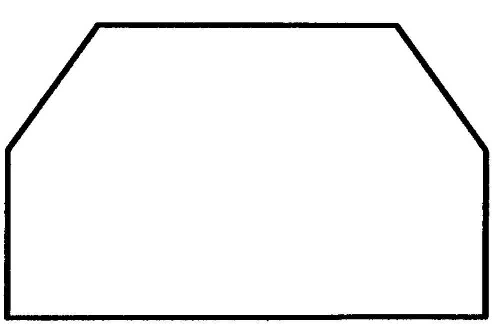

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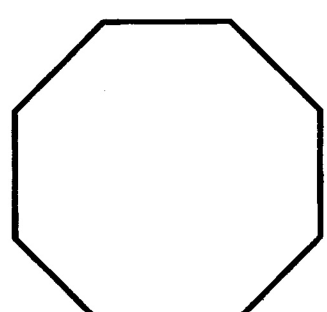

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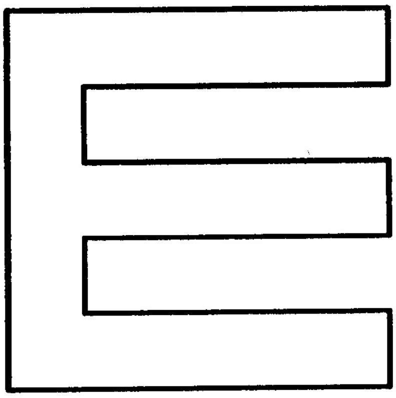

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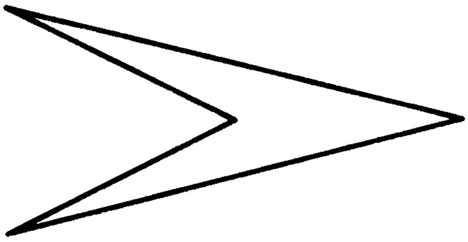

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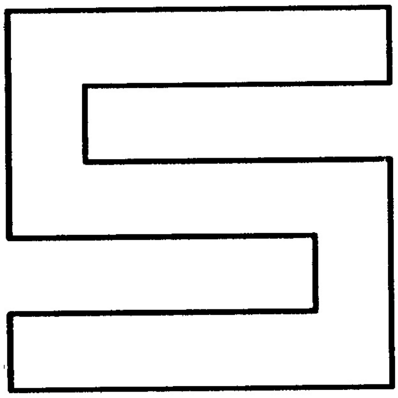

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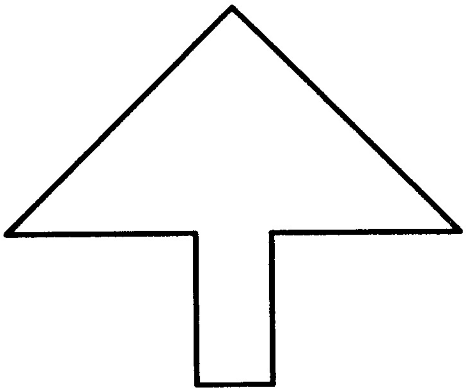

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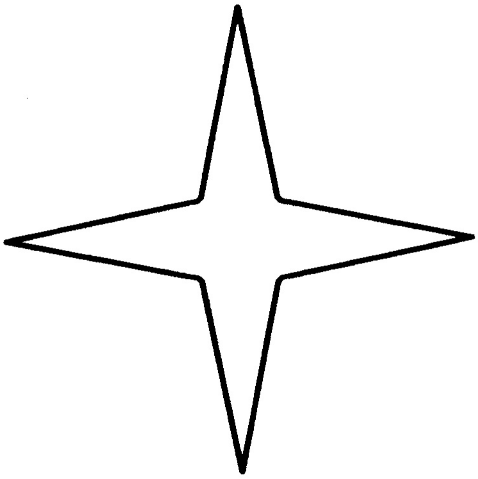

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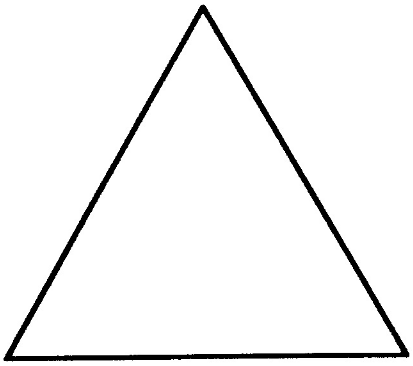

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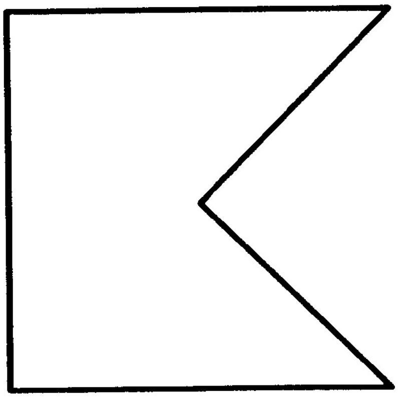

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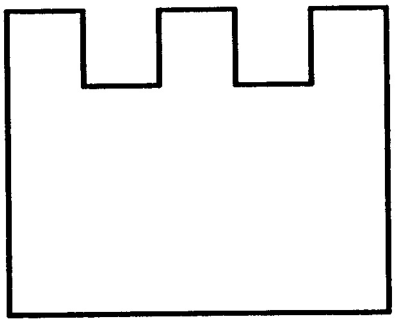

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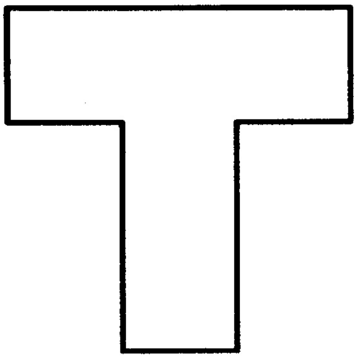

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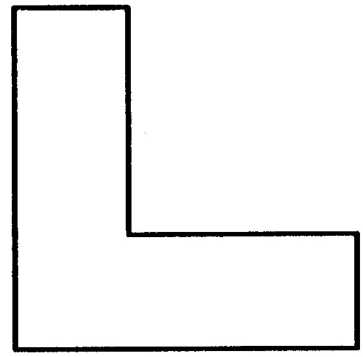

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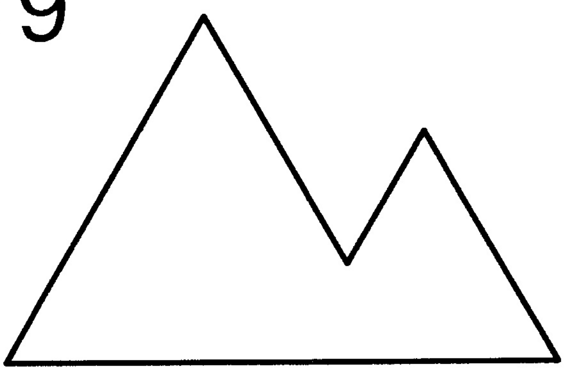

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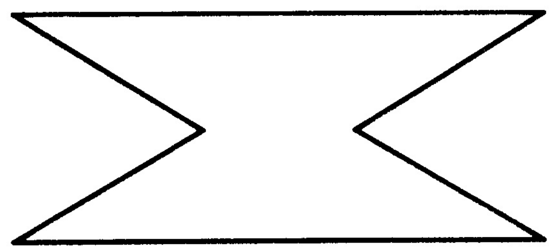

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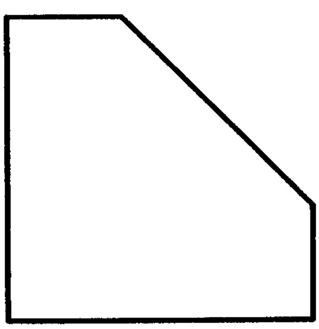

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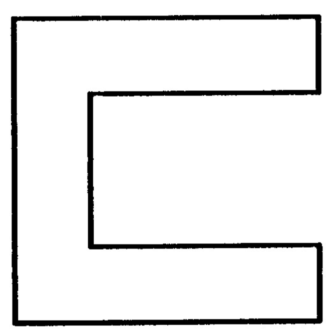

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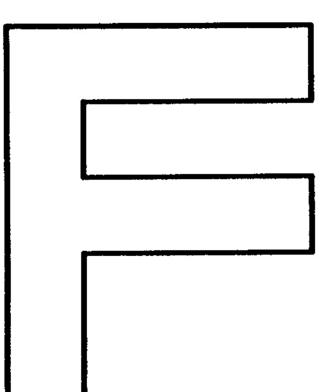

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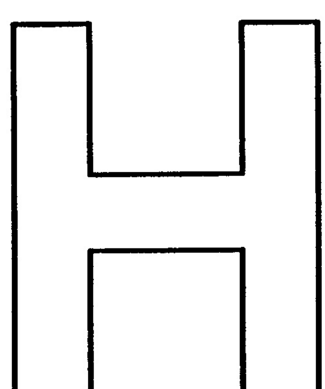

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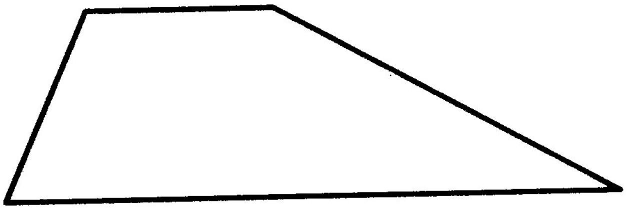

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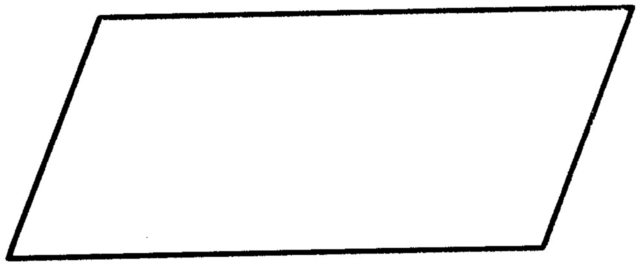

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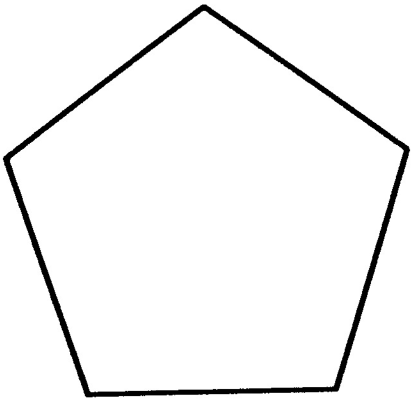

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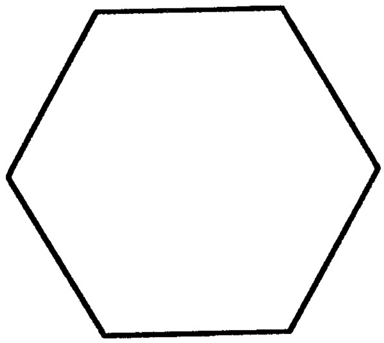

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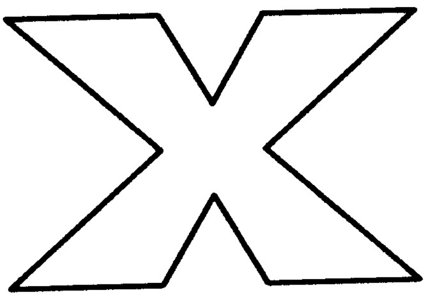

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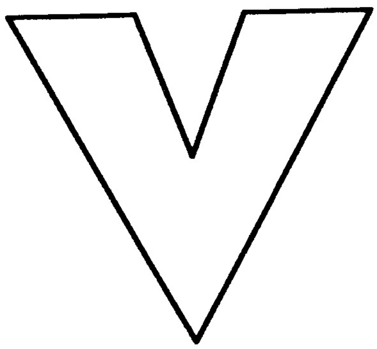

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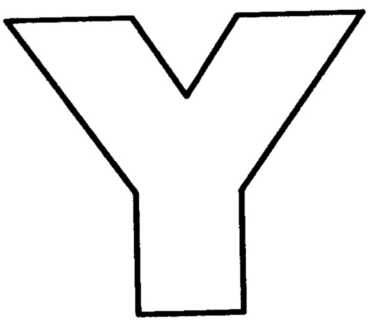

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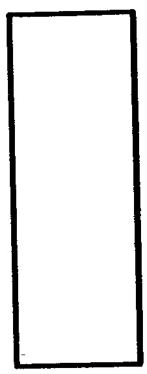

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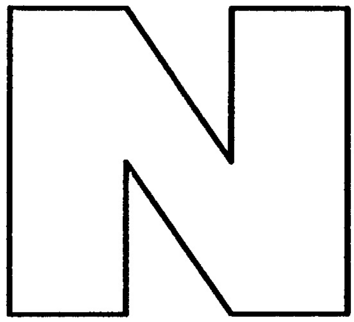

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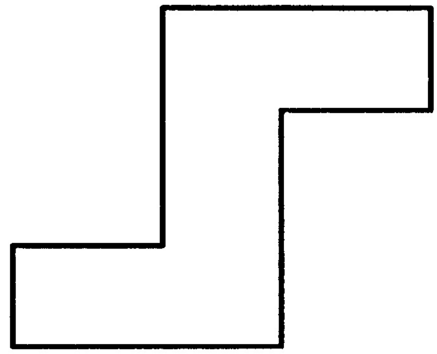

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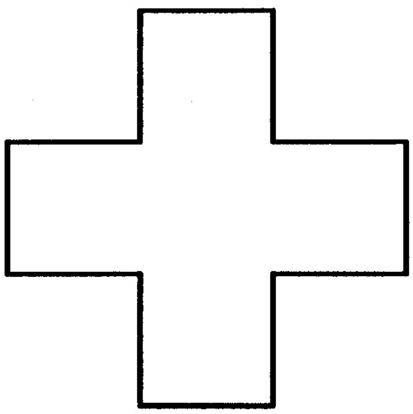

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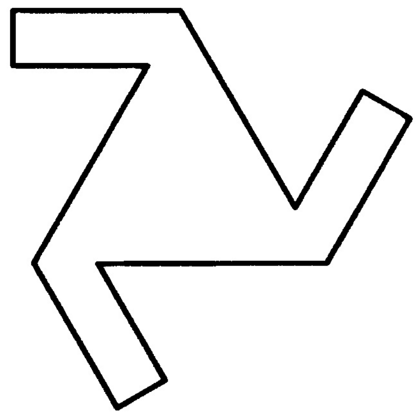

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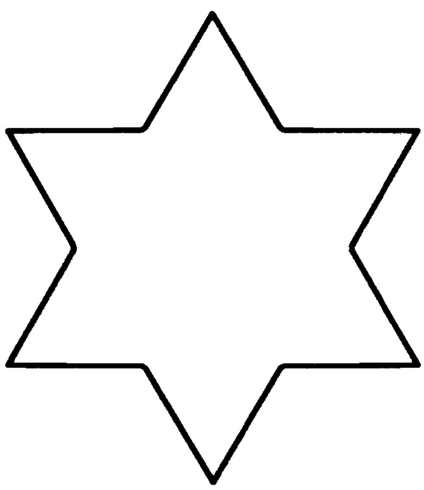

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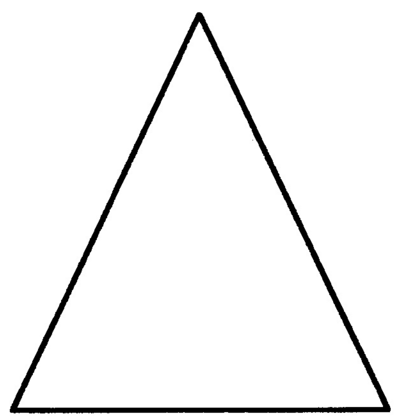

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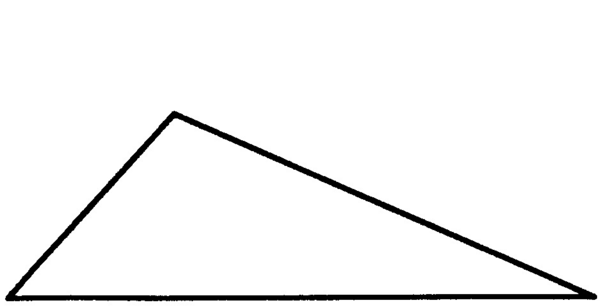

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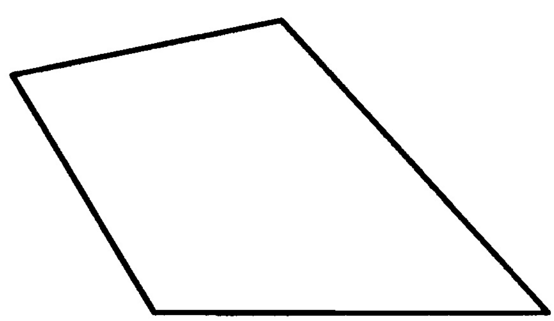

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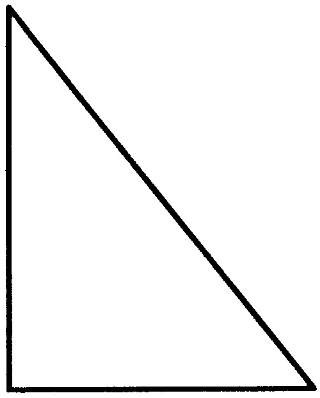

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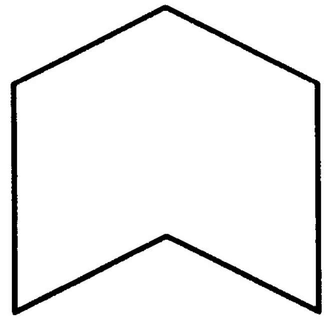

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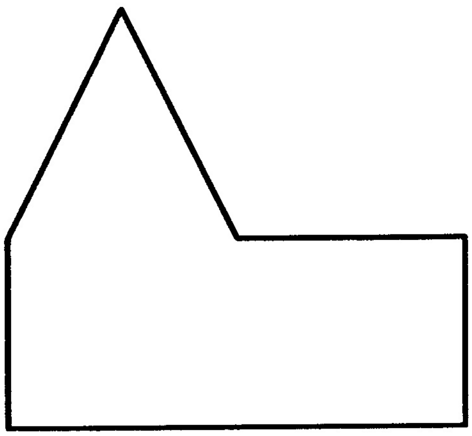

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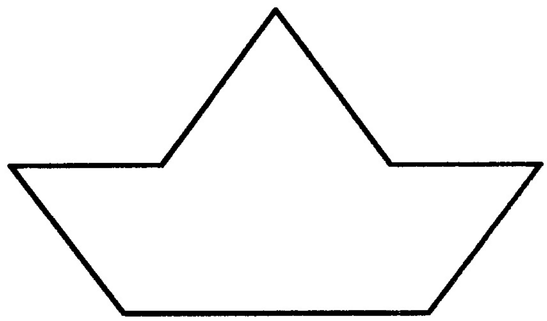

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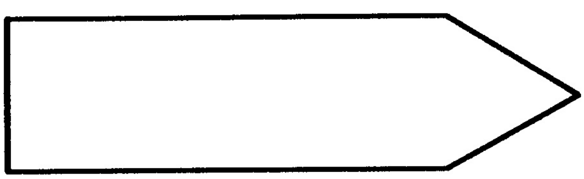

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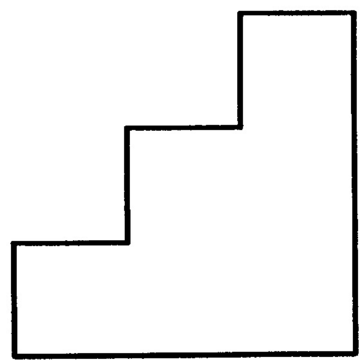

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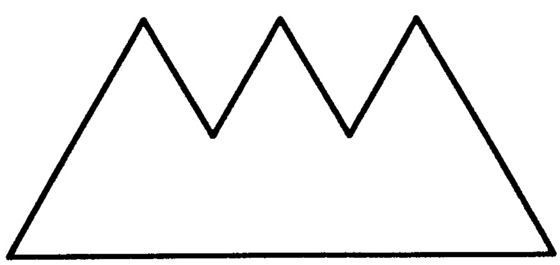

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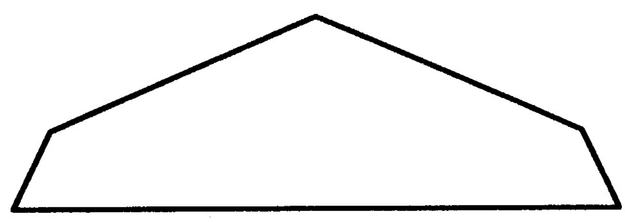

No.	Name	Sides	Bilateral	Rotational	Rt Angle	Vert/Horiz	Outline	Tessellates
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
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48								

Record Sheet

For every shape, record the details of the questions that you asked of the database

Investigations:

1. If a shape is made up entirely of vertical and horizontal lines, does it always contain a right angle?

2. If a shape is made up entirely of vertical and horizontal lines, and has more than four sides, is it always concave?

3. On a concave shape, in which every side is either vertical or horizontal, what is the least number of sides the shape must have?

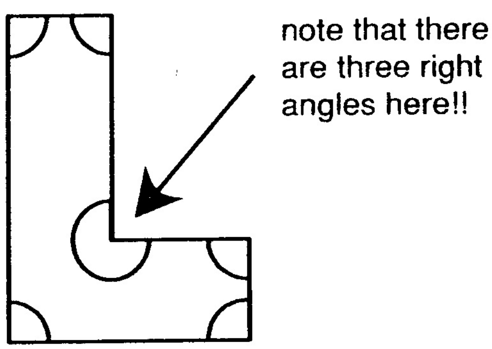
4. Is it possible to have a shape with two lines of bilateral symmetry, but rotational symmetry of order 1?

5. If a shape has an order of rotational symmetry that is even, does it follow that it must have at least one line of bilateral symmetry?
(hint: search for 'Rotational' is equal to 2 OR 'Rotational' is equal to 4 OR ......)

6. If a shape has rotational symmetry of order 2 or more, does it mean that all the sides must be the same length?
(hint: search on 'Rotational' is greater than 1 and then list the shapes - you will then need to look back at the cards to find the answer! To disprove the hypothesis, just find one exception to the rule.)

7. If a shape has 2 or more lines of bilateral symmetry, must one of these be horizontal?
(hint: in this example you will also need to look at the cards - otherwise how will you know whether the lines of bilateral symmetry are horizontal?)

8. Consider only the shapes where all the sides are vertical or horizontal. Is there any relationship between the number of sides of these shapes and the number of internal right angles? Does the same conclusion apply to all the shapes?
(hint: you will need to refer to the cards to work out the number of internal right angles in each shape)



9. Using the same shapes as in question 8, is there any relationship between the number of internal right angles in a shape and whether or not it is concave?

10. Considering all shapes on the database, is there any relationship between concave shapes and reflex angles (those which are greater than 180 degrees)? If so, what is it?

11. What do you notice about the tessellation property of all the triangles? Is the same true of all the four sided shapes? How about for the five sided shapes?
(hint: try three separate searches in order to answer the question)

12. What do you notice about the number of lines of bilateral symmetry on shapes with an odd number of sides?
(hint: search for 'Sides' is equal to 3 OR 'Sides' is equal to 5 OR ....)

13. The database includes descriptions of equivalent shapes (those with identical properties). Can you find a set of 3 shapes, all with identical properties? Are there any other sets of 2 or 3 equivalent shapes?
(hint: the answer to this investigation should be apparent from when you tried to identify them!).

'Puzzling Shapes' and the National Curriculum

From the Orders for Mathematics:

* identify and obtain information necessary to solve problems	(AT1, lv 4)
* use mathematical terms to describe common 2-D and 3-D shapes	(AT4, lv 2)
* recognise reflective symmetry	(AT4, lv 3)
* sort shapes using mathematical criteria and give reasons	(AT4, lv 3)
* recognise rotational symmetry	(AT4, lv 4)
* use properties of shape to justify explanations	(AT4, lv 5)
* access information in a simple database	(AT5, lv 3)
* interrogate and interpret data in a computer database	(AT5, lv 4)
* use a computer database to draw conclusions	(AT5, lv 5)

From the Orders for Technology:

* use information technology for the storage & retrieval of information	(AT5, lv2)
* collect information and enter it in a database (whose structure may have been prepared in advance), and to select and retrieve information from the database	(AT5, lv 3)
* amend and add to information in an existing database, to check its plausibility and interrogate it	(AT5, lv 4)
* use information technology to explore patterns and relationships, and to form and test simple hypotheses	(AT4, lv 5)

'Puzzling Shapes' Answer Sheet

No.	Name	Sides	Bilateral	Rotational	Rt Angle	Vert/Horiz	Outline	Tessellates
1	Window	4	4	4	yes	yes	convex	yes
2	Tank Top	10	1	1	yes	no	concave	no
3	Truck	6	0	1	yes	yes	concave	yes
4	Circus stool	4	1	1	no	no	convex	yes
5	Tile	4	2	2	no	no	convex	yes
6	Horse	11	1	1	yes	no	concave	no
7	Fire Station	6	1	1	yes	no	convex	no
8	Coin	8	8	8	no	no	convex	no
9	Comb	12	1	1	yes	yes	concave	yes
10	Spear	4	1	1	no	no	concave	yes
11	Snake	12	0	2	yes	yes	concave	yes
12	Larch	7	1	1	yes	no	concave	no
13	Badge	8	4	4	no	no	concave	no
14	Wigwam	3	3	3	no	no	convex	yes
15	Jaws	5	1	1	yes	no	concave	yes
16	Fort	12	1	1	yes	yes	concave	yes
17	Table	8	1	1	yes	yes	concave	yes
18	Elbow	6	1	1	yes	yes	concave	yes
19	Rockies	5	0	1	no	no	concave	no
20	Diablo	6	2	2	no	no	concave	yes
21	Slide	5	1	1	yes	no	convex	yes
22	Chair	8	1	1	yes	yes	concave	yes
23	Wrench	10	0	1	yes	yes	concave	yes
24	Ladder	12	2	2	yes	yes	concave	yes
25	Racer	4	0	1	no	no	convex	yes
26	Carpet	4	0	2	no	no	convex	yes
27	Head	5	5	5	no	no	convex	no
28	Table mat	6	6	6	no	no	convex	yes
29	Stretch	12	2	2	no	no	concave	no
30	Life Jacket	6	1	1	no	no	concave	no
31	Catapult	9	1	1	yes	no	concave	no
32	Mirror	4	2	2	yes	yes	convex	yes
33	Bendy	10	0	2	yes	no	concave	no
34	Step	8	0	2	yes	yes	concave	yes
35	Keyhole	12	4	4	yes	yes	concave	yes
36	Spinner	12	0	3	yes	no	concave	no
37	Night Sky	12	6	6	no	no	concave	no
38	Hat	3	1	1	no	no	convex	yes
39	Pinnacle	3	0	1	no	no	convex	yes
40	Cheese	4	1	1	no	no	convex	yes
41	Wedge	3	0	1	yes	no	convex	yes
42	Space Ship	6	1	1	no	no	concave	yes
43	Abbey	6	0	1	yes	no	concave	no
44	Tug	7	1	1	no	no	concave	yes
45	Crayon	5	1	1	yes	no	convex	yes
46	Stairs	8	1	1	yes	yes	concave	yes
47	Crown	7	1	1	no	no	concave	yes
48	Jelly	5	1	1	no	no	convex	no

Puzzling Shapes - Answers

Q1. Yes.

Q2. Yes. A search of the database gives 12 shapes, all of which are concave.

Vert/Horiz *is the same as* ***yes*** AND **Sides** *is greater than* **4**

Q3. This gives rise to the same set of shapes as the previous question, and the evidence is that the least number of sides is 6.

Q4. All the shapes on the database with 2 lines of bilateral symmetry also have rotational symmetry of order 2.

Q5. There are 4 shapes with rotational symmetry of order 2, but no lines of bilateral symmetry (Bendy, Carpet, Snake and Step).

Q6. No. For example, one exception to the rule is Bendy.

Q7. Not necessarily. An example from the shapes as given is 'Head'. However, consider a square and rotate it slightly. It still retains the same lines of bilateral symmetry, but they are no longer vertical and horizontal.

Q8.

Shape	Sides	Rt angles	Shape	Sides	Rt angles
chair	8	12	comb	12	20
elbow	6	8	fort	12	20
keyhole	12	20	ladder	12	20
mirror	4	4	snake	12	20
stairs	8	12	step	8	12
table	8	12	truck	6	8
window	4	4	wrench	10	16

The number of internal right angles is double the number of sides minus 4 (ie. (2n - 4) right angles).

Q9. If the number of internal right angles is 6 or more, then the shape is concave.

Q10. All concave shapes must have at least one reflex angle.

Q11. All triangles tessellate, as do all quadrilaterals. The same is not true for all pentagons (5 sided figures)

Q12. There are 15 shapes with an odd number of sides. Three of these shapes don't have any lines of bilateral symmetry. Of the other 12, the number of lines of bilateral symmetry is always odd (in fact in 10 of the cases there is just one line).

Q13. One example is that the fort is equivalent to the comb. Less obviously, the chair, table and stairs are all equivalent in terms of the field descriptions given.

Fascinating Numbers

"Suddenly discovering an interesting pattern or relationship in mathematics and conjecturing that perhaps no one since time began has ever known that fact, however trivial it might be, can have the same intellectual satisfaction as finding a vein of gold that no one else has ever seen." Boyd Henry, 1987

This database is designed to encourage pupils to play with numbers, to discover hidden properties and to conjecture about plausible connections. It is suitable for pupils at the upper end of the primary school and for those in secondary education. The aim is to guide the individual into the endless wonderland of patterns inherent in our number system. They may be inspired to develop further enquiries and to start asking "What happens if ....?" The following suggestions provide starting points for a number of mathematical explorations. As well as exploring, children are gaining new number knowledge and developing problem solving strategies. Collaborative group work is essential to this process as this enables children to discuss their ideas with their peers and to articulate their reasoning. It should also help them to develop systematic ways of recording the results of their investigations. Teachers will be active partners in these explorations, from co-discoverers on the one hand to the providers of extension ideas on the other. If we can engender an enthusiasm for 'number' in young children, this will undoubtedly contribute to the acquisition of a positive attitude to mathematics, so helpful in later life.

In using these materials, children will be:
a) searching a database for evidence to help answer a particular question;
b) recording the information in an appropriate manner;
c) looking for patterns and extending these patterns;
d) making conjectures concerning possible relationships between numbers;
e) testing these conjectures by further exploration;
f) working and communicating with others.

Before starting on the suggested activities, pupils need to know how to interrogate a database. If they are not familiar with this, they should begin by using one of the other activities in this package. These exercises are the most sophisticated of the 4 in this package in terms of mathematical thinking.

Fascinating Numbers and the National Curriculum.

The information in this database is designed to address a number of Attainment Targets in both the Maths and Technology Orders.

From the Mathematics Orders:

*	investigate general statements by trying out some examples	(AT1, lv3)
*	i) give some justification for their solutions to problems	(AT1, lv4)
	ii) make generalisations	
*	make a generalisation and test it	(AT1, lv5)
*	pose their own questions or design a task in a given context	(AT1, lv6)
*	examine critically the mathematical presentation of information	(AT1, lv6)

*	make a generalisation, giving some degree of justification	(AT1, lv6)
*	solve problems involving muliplication or division	(AT2, lv3)
*	solve number problems with the aid of a calculator, interpreting the display	(AT2, lv4)
*	explore number patterns	(AT3, lv2)
*	use pattern in number when doing mental calculations	(AT3, lv3)
*	make general statements about patterns	(AT3, lv4)
*	explore number patterns using computer facilities or otherwise	(AT3, lv6)
*	interpret relevant data which has been collected	(AT5, lv2)
*	access information in a simple database	(AT5, lv3)
*	construct and interpret statistical diagrams	(AT5, lv3)
*	interrogate and interpret data in a computer database	(AT5, lv4)
*	use a computer database to draw conclusions	(AT5, lv5)

From the Technology Orders

*	use information technology for the storage and retrieval of information	(AT5, lv2)
*	amend and add to information in an existing database, to check its plausibility and interrogate it	(AT5, lv4)
*	use information technology to explore patterns and relationships, and to form and test simple hypotheses	(AT4, lv5)

The database structure

The database is called 'Number' and includes data on the first 900 numbers. A typical record is shown below and the accompanying 'field definition' sheet describes the different field names. If you consider that the database contains too many numbers for the ability of the pupils you are working with, it is possible to search the database on, for example, 'Number *is less than* 256' and then save this as a separate file. It is also possible to save a subset of the available fields (by deleting the unwanted ones). If you wish to combine the two 'factors' fields into one, this can be done but it will entail an element of manual editing.

File name: **Number** - an example record, for number 144:

Number	144
Factors<=50	1 2 3 4 6 8 9 12 16 18 24 36 48
Factors>50	72 144
Number of factors	15
Prime factors	2 2 2 2 3 3
Series	Squ (could be Tri, Squ, Pen, Hex, Fib, Dbl or Cub)
Palindromic	No
Prime	No
Digital Root	9

Notes:

a) In the case of numerical lists of numbers (eg. factors<=50 or prime factors), the first number in the list has been preceded by a space and the last number in the list has been followed by a space. This is to enable the user to search for a particular number in the list by 'including' a space either side of it. For example, if you wished to find out whether or not a number is a multiple of 5, you may decide to do this by looking to see whether the **'factors<=50'** field includes a 5. The search criteria would be:
Factors<=50 ***'includes'*** *_5_* (where the _ indicates that you have pressed the spacebar)
If you had failed to put in the spaces, the search would also pick out numbers like 15 or 52 (and many others!)

b) Remember that if you answer any of the questions by sorting the database, you will have reshuffled the order of the numbers. They will remain reshuffled until you sort them again on the **'number'** field, smallest first.

(The Nimbus version of Grass included on the disc makes it apparent when spaces have been typed in the search criteria, by printing black on white and hence clearly showing whether a space has been inserted.)

Starting points for investigating the data

The worksheets, 'Investigations with Number', give some introductory activities for using the database. A few comments on the different activities are given below. The answers are provided on a separate sheet. With some questions pupils will start by using the data contained in the database, while with others they may work manually and simply use the database to check their answers.

* The first 3 questions involve predictive skills, using the database to check the answers graphically.

* This is followed by a number of questions (4 - 12) that involve a simple search to provide some initial data, and then the application of observation and reasoning skills to make certain generalisations.

* Questions 13 & 14 encourage children to look at number series and to work out the number patterns that enable the series to be extended.

* Questions 15 - 17 focus on the digital root field and use this data to develop tests of divisibility.

* The next set of questions (18 - 22) return to the idea of number series and the underlying patterns, but in these cases they involve a 'complex' search - that is the need to search the database on at least two criteria.

* The next set of questions (23 - 26) are designed to encourage the pupils to hypothesise and make generalisations. In general the database only provides the starting information as the pupils then resort to paper and pencil (or a spreadsheet, see below) to provide further calculations from which to draw their generalisations.

* The concluding questions (27 - 30) involve number puzzles where the pupils act as number detectives to deduce the most appropriate answer. Pupils should be encouraged to produce questions and queries of their own for their peers to solve. Such questions as 'I'm thinking of a number ...' are well within the invention of young pupils.

The 'investigation' sheets provide starting points. Teachers can provide additional investigations for the pupils to explore, or the pupils could provide exciting challenges for each other.
In some cases pupils will be familiar with the use of spreadsheets. They may decide that the spreadsheet is a more appropriate tool with which to answer some of these questions. They should be encouraged to use whichever tool they think is the most appropriate. For example, the spreadsheet can be used to generate lists of triangular, square or fibonacci numbers and to examine the difference between adjacent terms in these series.

Further Reading.

Experiments with Patterns in Mathematics by Boyd Henry. Dale Seymour Publications, 1987. ISBN 0-86651-346-9
Number Skills in Problem Solving by Ken Milton. Archimedes Press Pty. Ltd, 1985. ISBN 0-949348-03-1

The Numbers Database

An example of a typical record:

Number	144
Number of factors	15
Factor<=50	1 2 3 4 6 8 9 12 16 18 24 36 48
Factors>50	72 144
Prime factors	2 2 2 2 3 3
Series	Squ *(or Tri, Squ, Pen, Hex, Fib, Dbl or Cub)*
Prime	No
Palindromic	No
Digital Root	9

Definition of Field Names:

Factors<=50

A factor is a number which divides exactly into the original number, without leaving a remainder. Those factors less than or equal to 50 are included in this field. It will always include the number 1 and the number itself (if it is less than or equal to 50). In the example given above there are 13 numbers all less than 50 that divide into 144 exactly. By contrast, in the case of 143 there would only be 3 numbers (1, 11 and 13).

Factors>50

Those factors greater than 50 are recorded in this field. It will also include the number itself if it is over 50.
NB The factors have been separated between two fields because, in the case of the big numbers, there are sometimes too many factors to be stored in one field.

Number of factors

The total of the number of factors (in both fields)

Prime factors

These are calculated by breaking the number down into its smallest prime factors. For example, in the case of 144:

$144 = 24 \times 6$
$144 = 12 \times 2 \times 6$ (because $24 = 12 \times 2$)
$144 = 6 \times 2 \times 2 \times 6$ (because $12 = 6 \times 2$)
$144 = 3 \times 2 \times 2 \times 2 \times 3 \times 2$ (because $6 = 3 \times 2$)

Hence the prime factors of 144, arranged in order of size, are 2 2 2 2 3 3

Series

This identifies which series the number is part of. A number may be found in more than one series. The series included are:

Triangular	(Tri)	see diagram
Square	(Squ)	see diagram
Pentagonal	(Pen)	see diagram
Hexagonal	(Hex)	see diagram
Doubling	(Dbl)	doubling numbers assume a starting point of 1 and then double (ie. 1 2 4 8 .....)
Fibonacci	(Fib)	this sequence (1 1 2 3 5 8 13 21 34 ....) is where each number is the sum of the two preceeding numbers.
Cubic	(Cub)	cubic numbers are those that can be expressed in the form n x n x n (or n^3), such as 64 because it can be written as 4 x 4 x 4.

Palindromic

These numbers read the same forwards and backwards (eg. 141 or 454).

Prime numbers

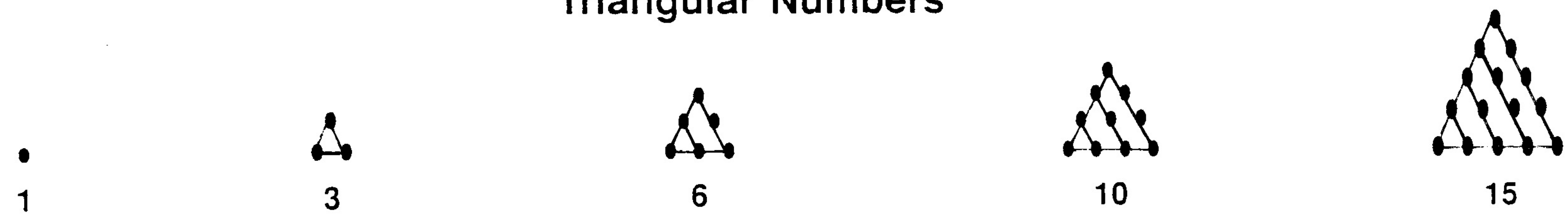
Those numbers, such as 17, that only have two factors, namely 1 and the number itself. No other number will divide exactly into the original number.

Digital root

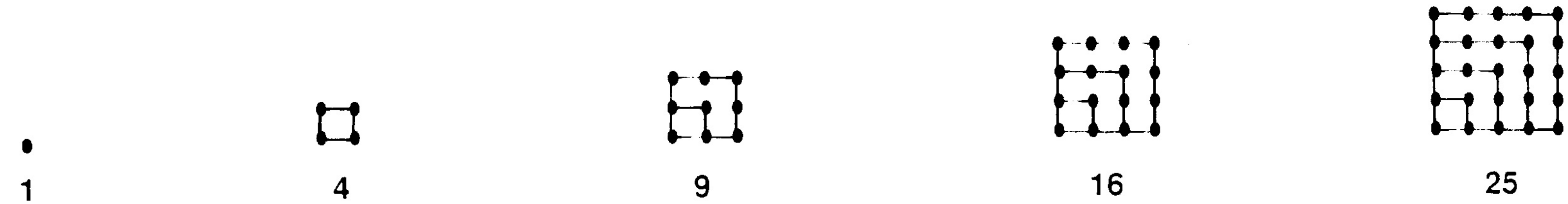
The digital root of a number is the sum of the digits of a number, repeated if necessary, until you get down to a single figure. For example:

189 = 1 + 8 + 9 = 18, 18 = 1 + 8 = 9
Hence 9 is the digital root of 189

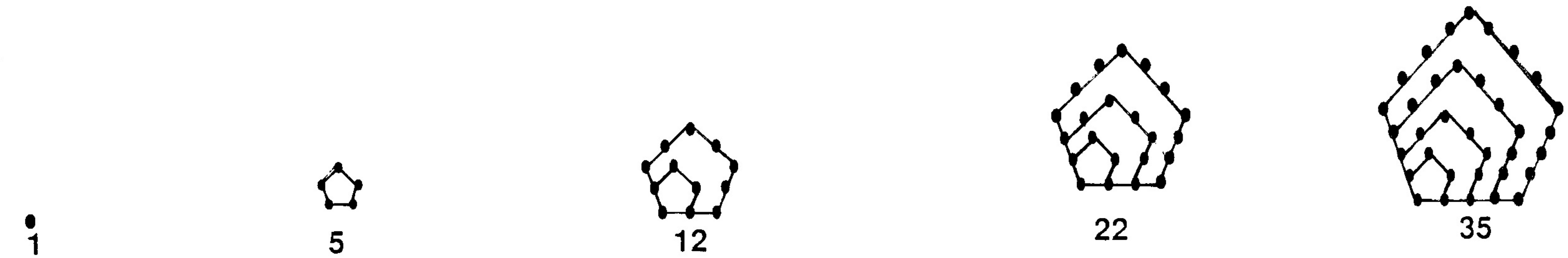
Triangular Numbers



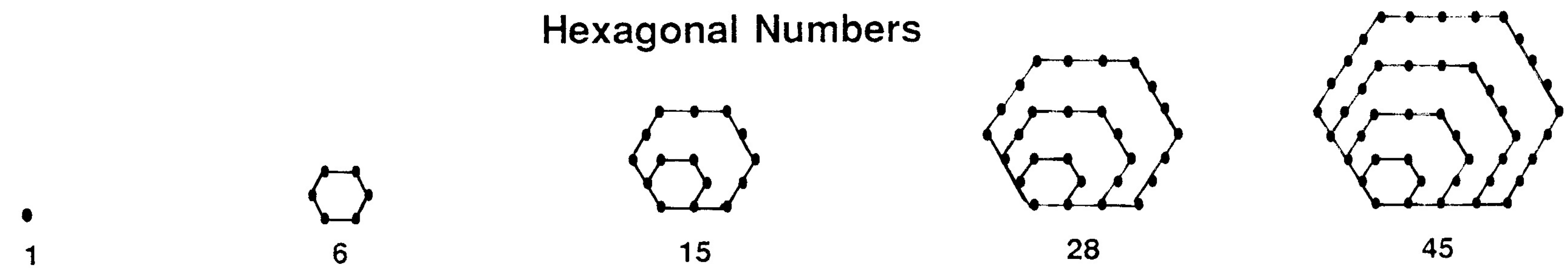
Square Numbers



Pentagonal Numbers



Hexagonal Numbers



Investigations with 'Number'

Primes - a rare find?

Question 1.

Do prime numbers occur more frequently than other numbers (ie. those which are not prime)?

Hint: Are even numbers prime? How often do you get an even number?

Method: Use the database to check your answer. Select the 'graphs and figures' option on Grass and choose the pie chart option. Display a comparison between those numbers which are prime and those which are not. Which occur the most often?

The same either way.

Question 2.

Do palindromic numbers occur more frequently than other numbers (ie. those which are not palindromic)?

Hint: Think about 141, for example. How long is it before the next palindromic number occurs?

Method: Use the database to check your answer. Display a comparison between those numbers which are palindromic and those which are not. Which occur the most often?

Frequency of factors.

Question 3.

If the number of factors of all the 900 numbers are displayed in the form of a histogram, what shape would the graph be?

Hint: Do you think that the majority of numbers have many factors or just a few? How many factors do the prime numbers have?

Method: Check your answer by drawing a histogram from the information in the field called 'number of factors'.

Friends of 72.

Question 4.

What are the factors of 72? Write down two numbers which divide into 72 exactly and another two numbers which don't.

Method: Search on **number** *is equal to* 72, and then display the whole of the record.

Prime endings

Question 5.

List the prime numbers. Look at the units figures. Which digits never seem to appear? Hence, which of these numbers is instantly recognisable as *not* prime:

175 256 483 679

Hint: Remember that the only information recorded in the 'prime' field is either yes or no.

Prime factors.

Question 6.

List the prime factors of 90. You should find that there are four prime factors. Multiply the numbers together. What do you notice? From this information, can you write down just two numbers which when multiplied together will give 90? Can you find another two?

Now do the same for 120. Then write down three numbers which when multipled together give 120. How many ways are there to do this?

Investigations with 'Number'

Prime factors.

Question 7.

What are the prime factors of:

i)	54	ii)	208
iii)	456	iv)	720?

Work them out by hand before using the database to check your answer?

Hint: Use a calculator to help you

Guessing at factors.

Question 8.

Use the database to list all the factors of 108 and compare these with all the factors of 216. What do you notice. List all the factors of 240 and hence write down the factors of 720.

Hint: Remember that it is necessary to look at two fields in order to get a complete list of factors

Method: In order to make comparisons, you will need to copy down the factors of 108 on paper so that you can compare them with those of 216.

Rich in factors.

Question 9.

Use the database to find which number in the list has the most factors. Write down a number, not in the database, that has at least one more factor.

Method: To find the 'most' it is easiest to identify the field you are examining (in this case it is **number of factors**) and then to sort the data in ascending or descending order.

Cubic numbers.

Question 10.

What do you notice about the prime factors of cubic numbers? Using this information, predict the next two cubic numbers that occur after 900.

Hint: Be aware that a number like 216 is 6 x 6 x 6, but it will appear as 2 2 2 3 3 3.

Method: This will involve searching the database for cubic numbers and displaying each of these together with their prime factors.

Numbers that are doubling.

Question 11.

What do you notice about the prime factors of numbers in the doubling series?

Method: Search to see whether **series** *includes* **Dbl**. List these numbers together with their prime factors and the answer should then be apparent.

Symmetrical numbers?

Question 12.

Use the database to display the palindromic numbers. Study the pattern of the answers. Write down the next three palindromic numbers that occur after 900.

Hint: Remember that the information in the **palindromic** field is recorded as either yes or no. Rather than look at all the palindromic numbers, you may choose only to look at those greater than 700.

Investigations with 'Number'

Triangular numbers.

Question 13.
From the database find all the triangular numbers and list them. Calculate the rate at which the gap grows between successive terms in the triangular number series.
Look at your list of triangular numbers. Add up successive pairs. For example:
1 3 6 10 ....
1+3 3+6 6+10 .....
What do you notice about the results?

Hence you should be able to explain 2 ways of working out the next terms in any triangular number series. Apply your theory to predict the next two terms in the series bigger than 900?

Hexagonal numbers.

Question 14.

From the database find all the hexagonal numbers and list them. Calculate the intervals between successive terms and then work out the next two hexagonal numbers bigger than 900.

Method: This is similar to the previous question. You will need to find whether **series** *includes* **Hex**.

Merry multiples of 5.

Question 15.
Search the database for all the numbers with a factor of 5. These numbers are also called 'multiples' of 5. If a number is a multiple of 5, what do you notice about its units digit? Does this give you an easy test for divisibility by 5? Which of these numbers is divisible by 5:
69 455 5433 6780 12765?
If the multiples of 5 are listed in order, is there a pattern in the corresponding digital roots?

Method: Search for **factors < 50** *includes* **5** (and remember to include a 'space' either side of the 5). After carrying out the search, display the *numbers*, *factors < 50* and the *digital root* fields.

Multiples of 3.

Question 16.
Search the database for all numbers with 3 as a factor; these numbers are called multiples of 3. If a number is a multiple of 3, what do you notice about its digital root? Does this give you an easy test for divisibilty by 3? Which of these numbers is divisible by 3:
69 455 5433 12765?
Using this information, and that about multiples of 5, can you tell which of the four numbers above is also a multiple of 15?
Hint: If a number is divisible by 15, it must also be divisible by both 3 and 5.
Method: If you are searching for **factors < 50** *includes* **3**, remember to include a space either side of the 3.

Other tests of divisibility.

Question 17.

Can you discover any other tests of divisibility? Can you see a pattern in the digital roots of multiples of other numbers?

Hint: Having found a rule for multiples of 3 and 5, you should be able to find rules for multiples of 2, 9 and 10. It is also possible to determine a rule for multiples of 4 and 6. Sometimes the clue lies in the units figure of the multiple (as with multiples of 2) and sometimes in the digital root (as with multiples of 9).

Back to triangular numbers.

Question 18.

How many triangular numbers are there less than 256?

Method: Firstly you will need to search for the triangular numbers (does **series** *include* **Tri** ?) and then *narrow down* the search to find those which are less than 256. The answer is the number of records that are found that satisfy both criteria.

Investigations with 'Number'

Even more triangular numbers.

Question 19.

How many even numbers are there in the triangular number series? Is there a pattern as to how often they occur?

Method: Search to see whether **series** *includes* **Tri** and make a list of these numbers on paper. Now think how you can identify even numbers and then *narrow down* the search. Compare the list of these answers with the full list of triangular numbers that you have written on paper (or printed out).

Do primes get harder to find ?

Question 20.

How many prime numbers are there in the database?
How many of these occur within the first hundred numbers? How many occur within the second hundred numbers? How many occur within the third hundred numbers?
Do you conclude that prime numbers become rarer or more frequent as the numbers get bigger?

Method: To find how many primes occur within the second hundred numbers, for example, search for:

	Number	*is greater than*	**99**
AND	**Number**	*is less than*	**200**
AND	**Prime**	*is the same as*	**yes**

Fibonacci numbers.

Question 21.

How many Fibonacci numbers are also odd? Is there a pattern here?

Hint: Remember that the Fibonacci series really starts with 1 1 2 3 5 (ie. a double 1 at the start).

Method: Search to see whether **series** *includes* **Fib** and make a list of these numbers on paper. Now think how you can identify odd numbers and then *narrow down* the search. Compare this list of answers with the full list of triangular numbers you have written on paper.

More work on number series.

Question 22.

Which numbers are in both the triangular number series and the hexagonal number series? Can you see a pattern here?
Which number, other than 1, occurs in the triangular and square number series?

Hint: In order to see a pattern you will need to list the triangular numbers before narrowing down the search to find those which are also hexagonal.

Investigations.

Question 23.
Write down, in order, the first 25 multiples of 4. Add 1 to each number and list these opposite. For example

4	5
8	9
12	13

Each number in the right hand list is said to be the square of an odd number or a prime number. Is this claim true? If it is false, give the first example for which it does not hold.
Hint: Although this question starts from information contained within the database, you will need to use paper and pencil (or a spreadsheet). Do the new results lead to any mathematical generalisations?
Method: Search for **factors < 50** *includes* **4** (and remember the space either side of the 4)

Investigations.

Question 24.
Investigate these 2 claims:
a) If a perfect square ends in a 6, the digit just before the 6 is always odd;
b) If a perfect square ends in a digit other than a six, the digit just before the last is always even.

Hint: After an initial search, you will need to use paper and pencil (or a spreadsheet). Do your results support these claims or have you found an example which proves them to be false?

Method: Search for **series** *includes* **Squ**. You cannot narrow down the search from here. You will need to observe the results and draw your conclusions.

Investigations with 'Number'

Investigations.

Question 25.
List the square numbers together with their digital roots. Investigate any patterns which may arise. There are a variety of outcomes; all you have to do is to justify the pattern that you have spotted. The same activity can be repeated with cubic numbers.

Hint: This question is open-ended. Try to find one pattern between the square numbers and their corresponding digital roots.

Method: Search for the square numbers (**series** *includes* **Squ**) and display these as a list together with their digital roots. Then look for a relationship.

Investigations.

Question 26.

Try to produce some lists of numbers that follow particular rules. Here are 3 lists to start on:
a) every number in the list must be odd
b) every number in the list must end with a 5
c) every number in the list must contain the digit 6
d) the tenth number in the list must be 860
e) the digital root of every number must be 4

Hint: In these examples there may be more than one way to get to a particular answer. Can you find more than one way to produce a particular list?

Puzzles - think of a number.

Question 27.

I think of a number and find that it has 12 factors. I add all the digits together once *and then again for a second time* before I arrive at the number 6. The number I am thinking of has more even prime factors than odd prime factors. What is the number?

Hint: This question involves a complex search, but will also involve some pencil and paper work to eliminate some of the numbers in the list until you get down to a final one.

Puzzles - think of a number.

Question 28.

I think of a number which has a digital root of 7 and is greater than 500. It is also a Fibonacci number. What number am I thinking of?

Method: This involves a complex search in three stages.

Writing about numbers.

Question 29.
This activity involves writing about numbers. Give your own personal opinion on:
a) Which is the most interesting number in the list and why?
b) Which is the most boring number in the list and why?

Hint: Views will differ. Some people will think a number is interesting because it can be found time and time again in different lists. Others will think it is interesting because it is rare and difficult to find. Perhaps the shape of the number interests you. What happens if you look at 818 in a mirror or turn it upside down? Give your reasons for your answers.

Puzzles - cryptic cricket.
Question 30.
The first 200 numbers are taken to see a dull cricket match. They may be needed to hang about on the scoreboard, but they try to creep away. All the even numbers leave at 5 o'clock to attend Evensong. They are shortly followed by all the prime numbers who are going on a special outing to see 'Prime Suspect'. The single digit numbers go to the creche - they are too young to appreciate cricket. All the palindromic numbers leave in case the ball cracks the mirror that is the very essence of their existence. Those with a large family (4 factors or more) go off visiting, as is customary on a Sunday teatime.
How many are left? They decide to confuse the scorer by adding themselves together. What is the overall total?

Investigations with 'Number'

A selection of answers

Q1. Out of the 900 numbers, 155 are prime (ie. 17%)

Q2. Out of the 900 numbers, 98 are palindromic (ie. 11%)

Q3. The graph produces 7 vertical bars which decrease in height moving from left to right. In fact just under half of the numbers have 0 to 4 factors. By contrast there are only 2 numbers with 30 - 34 factors; namely 720 and 840.

Q6. The prime factors of 120 are 2 2 2 3 5. Combinations of three numbers which multiplied together give 120 include:

2	2	30	2	4	15
4	6	5	4	3	10
8	3	5	2	5	12
10	6	2			

Q7.

54	2 3 3 3
208	2 2 2 2 13
456	2 2 2 3 19
720	2 2 2 2 3 3 5

Q8. To answer this question, search for **number** *is equal to* **108**, but then remember to display the number together with 'factors<=50' and 'factors>50'. In fact for 108 you should find 10 numbers in the former field and 2 in the latter, giving 12 factors in all, namely:
1 2 3 4 6 9 12 18 27 36 54 108
Note that the factors of 216 not only include 108 and 216, but also 8, 24 and 72.
The factors of 720 are the same as those for 240 but also include 9, 18, 36, 45, 72, 90, 144, 180, 360 and 720.

Q9. 840 has 32 factors. If you double the number (1680), it must have at least 33 factors - that is all the factors of 840 plus 1680 itself - but could there be more? What about 560? Is it a factor of 840? Is it a factor of 1680?

Q10. The prime factors of cubic numbers produce 3 identical numbers - with a little imagination! For example, 343 gives 7 7 7 - no problem! However 216 gives 2 2 2 3 3 3, which is another way of saying 6 6 6. Hence the biggest cubic number in the database is 729 (9x9x9), so the next two that occur after 900 are 1000 and 1331.

Q11. The only prime factors in the doubling series are 2s. The number of 2s increase by 1 for each successive term.

Q12. The largest palandromic number is 898. The next three are 909 919 and 929.

Q13. There are 41 triangular numbers in total. Adding together pairs of triangular numbers produces a list of square numbers.
The largest triangular numbers in the database are 741 780 820 861. The gaps between these numbers are 39 40 41 and hence the next gap will be 42, giving 903 as the next number in the series (861+42). Alternatively 820+861 is 1681 which is the square of 41. The square of 42 is 1764. If you subtract from this 861, it leaves 903!! After 903 the next term is 946.

Q14. The list should contain 21 numbers. The largest of which are 630 703 780 861. The gaps between these numbers are 73 77 81 and hence the next will be 85 giving 946 as the next hexagonal number in the series (861+85). After 946 the next term is 1035.

Q15. The units digit of multiples of 5 is always 0 or 5. There is a pattern in the corresponding digital roots, but it is slow to repeat; 5 1 6 2 7 3 8 4 9 and then 5 1 6 ....

Q16. The digital roots of multiples of 3 is always 3, 6 or 9.
If a number is divisible by 5 and also by 3, then it is divisible by 15 - in this case the answer is 12765.

Q17. A number is divisible by 2 if it ends in an even number
A number is divisible by 9 if its digital root is 9
A number is divisible by 6 if it ends in an even number and its digital root is 3, 6 or 9.

Q18. 22 triangular numbers are less than 256.

Q19. In the triangular number series up to 900 there are 20 even numbers. There is a pattern in how often they occur - two odd numbers are followed by two even numbers and then by two odd numbers and then two even numbers etc.

Q20. 155 prime numbers in the database.
In the first hundred numbers there are 26 primes
In the second hundred numbers there are 21 primes
In the third hundred numbers there are 16 primes
In the fourth hundred numbers there are 16 primes
You may conclude that primes get rarer, but you don't really have enough evidence here. For example, between 400 and 500 there are 17 primes, although between 800 and 900 there are only 15 primes.

Q21. In the Fibonacci series up to 900 there are 14 numbers, 9 of which are odd (excluding the extra 1 at the start). Remember that the Fibonacci series should start with 1 1 2 3 etc and hence the pattern is that two odd numbers are followed by one even number.

Q22. Every alternate number in the triangular number series is also in the hexagonal series. For example, the numbers underlined below are also hexagonal numbers.
<u>1</u> 3 <u>6</u> 10 <u>15</u> 21 <u>28</u>
The only number, other than 1, to occur in the triangular and square series is 36.

Q23. The claim is not true. The first exceptions are 21, 33 and 45. These are not prime, nor are they square numbers. However, if you consider those numbers that are the squares of odd numbers, they do occur with predictable frequency. Can you spot this?

Q24. Both of these statements are true for numbers up to 900. Are they true for all numbers?

Q25. Observations may include:
the number 7 always occurs in a pair
the digital roots only generate the numbers 1 4 7 9
the digital roots occur in a predictable pattern, namely:
1 4 9 7 7 9 4 1 9

Q27. The answer is 96

Q28. The answer is 610

Q29. There can be no definitive solutions to this question, although we will be delighted to recieve copies of interesting and original answers.

Q30. The answer is 243 (25 + 49 + 169)

Mathematics through Databases

"Mathematics through Databases" is a book which describes four different activities using pre-prepared databases. The activities are designed to foster the development of mathematical skills (including those of problem solving and data handling). The book is made up of four sections, each of which describes, in detail, the learning opportunities provided by exploring the associated datafile. Two activities are designed to introduce children to the idea of detective games using datafiles; one involves identifying the "real" Father Christmas while the other involves discovering which dog stole the sausages (among other things). The other two activities are designed for children with more experience of data handling. One activity focuses on the exploration of the attributes of a variety of geometrical shapes. The other activity offers a wide range of explorations into the world of number. Each investigation is well supported with activity ideas (ready for use by the children), illustrations of the characters concerned, disc based datafiles, and a range of suggestions for extension activities. Links with the National Curriculum are outlined. Teachers who are looking for a way in to datahandling for younger children will find the first two activities particularly appropriate. Those who are looking for a way to extend children's datahandling skills and to improve their mathematical understanding will find a wealth of potential in the final two activities. The book is accompanied by a disc containing all the data files - simply put the disc in the drive and you are ready to go.

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